18th Philippine Mathematical Olympiad

Qualifying Stage

17 October 2015

PART I. Choose the best answer. Each correct answer is worth two points.

1. If $(x+5)(2x-3)=0$, what are all the possible values of $2x-3$?

   (a) $\frac{13}{2}$ only (b) 0 and -13 (c) 0 and -5 (d) -5 and $\frac{3}{2}$

2. What is the sum of all the even positive divisors of 64?

   (a) 61 (b) 62 (c) 126 (d) 128

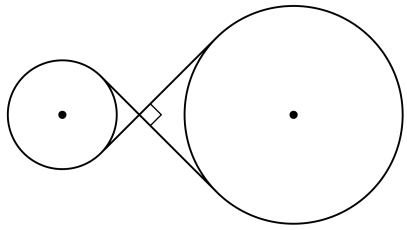
3. The measures of the angles of a quadrilateral are x, $x+10$, $x+30$, and $x+40$ degrees. What is the largest angle, in degrees?

   (a) 70 (b) 90 (c) 110 (d) 120

4. A ferris wheel with radius 12 meters rotates clockwise at the rate of 10° per second. Initially, Richard is sitting on the seat at the very top of the wheel, which has a height of 27 meters above the ground. How high above the ground will Richard be after the wheel has rotated for exactly one minute?

   (a) 6 meters
   (b) 9 meters
   (c) $(3+4\sqrt{3})$ meters
   (d) $(15-6\sqrt{3})$ meters

5. The two internal tangents of two non-overlapping circles of radii 2 cm and 4 cm intersect at right angles, as shown in the figure below.



   What is the distance between the centers of the circles?

   (a) $6\sqrt{2}$ cm (b) $6\sqrt{3}$ cm (c) 12 cm (d) $12\sqrt{2}$ cm

6. How many positive integers less than or equal to 2015 are divisible by 3, but are neither divisible by 5 nor by 7?

   (a) 455 (b) 461 (c) 506 (d) 512

7. Which of the following is the fifth largest divisor of the number $2,015,000,000$?

   (a) $125,937,500$ (b) $155,000,000$ (c) $201,500,000$ (d) $251,875,000$

8. What is the remainder when 25^{2015} is divided by 18?

   (a) 11 (b) 13 (c) 15 (d) 17

9. Triangles PQR and QRS, where $P \neq S$, are two right triangles sharing the hypotenuse QR. If C_1 is the circle passing through P, Q, and R, and C_2 is the circle passing through Q, R, and S, what can be said about $C_1 \cap C_2$?

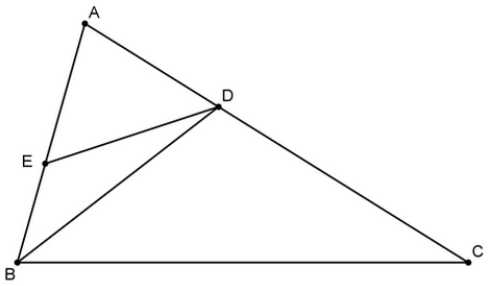
   (a) $C_1 \cap C_2$ consists of exactly one point.

   (b) $C_1 \cap C_2$ consists of exactly two distinct points.

   (c) $C_1 \cap C_2$ is empty.

   (d) $C_1 \cap C_2$ consists of infinitely many points.

10. In triangle ABC, BD is the angle bisector of $\angle ABC$. Moreover, $AB = BD$ and $AE = AD$. If $m\angle ACB = 36°$, determine $m\angle BDE$.



   (a) 24° (b) 18° (c) 15° (d) 12°

11. In how many ways can the letters of the word QUALIFYING be arranged such that the vowels are all in alphabetical order?

   (a) 24 (b) 151,200 (c) 181,400 (d) 3,628,800

12. When $n+5$ is divided by 4, the remainder is 3. When $n+4$ is divided by 5, the remainder is 4. What is the remainder when $n+6$ is divided by 20?

(a) 4 (b) 5 (c) 14 (d) 16

13. It is known that $\frac{2}{5}$ of chips scattered on the table are red. We remove a quarter of the chips from the table, and it is found $\frac{1}{4}$ of those removed are red. What fraction of the chips that remained on the table are red?

(a) $\frac{27}{80}$ (b) $\frac{9}{20}$ (c) $\frac{1}{16}$ (d) $\frac{1}{6}$

14. Mary wants to give 15 cookies to Amy, Bob and Charlie. How many ways can she distribute the cookies to them such that Amy must receive at least 5 cookies while Bob and Charlie must each receive at least 1?

(a) 40 (b) 45 (c) 50 (d) 55

15. The exterior of a cube of side length 3 cm is painted red, and is then divided into 27 unit cubes. If one of these cubes is randomly selected and then rolled, what is the probability that a red face comes up?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

PART II. Choose the best answer. Each correct answer is worth three points.

1. A married couple has PhP $50,000$ in their joint account. In anticipation of their upcoming anniversary, they agreed to split-up evenly and buy each one a gift. However, they agreed that they must have enough money left such that when combined, a minimum bank balance of PhP $5,000$ is maintained. If the couple individually bought gifts without regard to how much the other's gift will cost and each gift is randomly priced between PhP 0 to PhP $25,000$, what is the probability that they will be able to maintain the minimum required balance after buying the gifts?

(a) 2% (b) 20% (c) 80% (d) 98%

2. What is the difference between the largest and smallest real zeros of the function

$$f(x) = 2x^4 - 7x^3 + 2x^2 + 7x + 2?$$

(a) $\frac{5}{2}$ (b) $2\sqrt{2}$ (c) $1+\sqrt{2}$ (d) $\frac{3}{2}+\sqrt{2}$

3. Let S be the set of all points A on the circle $x^2 + (y-2)^2 = 1$ so that the tangent line at A has a non-negative y-intercept; then S is the union of one or more circular arcs. Find the total length of S.

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$ (c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{6}$

4. How many positive integers n make the expression

$$7^n + 7^3 + 2 \cdot 7^2$$

a perfect square?

(a) 0 (b) 1 (c) 3 (d) infinitely many

5. The numbers $1, 2, \ldots, 12$ have been arranged in a circle. In how many ways can five numbers be chosen from this arrangement so that no two adjacent numbers are selected?

(a) 15 (b) 36 (c) 180 (d) 396

6. How many (nonconstant) polynomial factors with leading coefficient 1, with the other coefficients possibly complex, does $x^{2015} + 18$ have?

(a) 2015
(b) $\sum_{k=1}^{2015} \binom{2015}{k}$
(c) 2^{2015}
(d) $\sum_{k=0}^{2014} 2015^k$

7. Let $a = 25^{12}$, $b = 16^{14}$, and $c = 11^{16}$. Which of the following is true?

(a) $b < a < c$ (b) $a < c < b$ (c) $c < a < b$ (d) $b < c < a$

8. How many integers x are there, where $100 \leq x \leq 2015$, and x is divisible by 3 or 8, but not by 6?

(a) 400 (b) 402 (c) 479 (d) 481

9. Find the 2015th digit in 122333444455555...

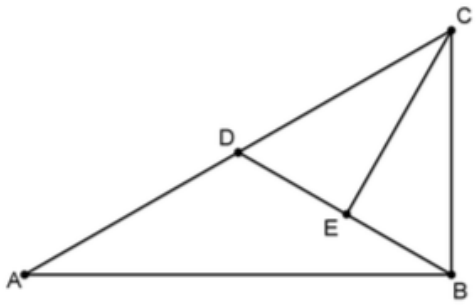
(a) 4 (b) 5 (c) 6 (d) 7

10. Find the sum of $\sum_{i=1}^{2015} \left\lfloor \frac{\sqrt{i}}{10} \right\rfloor$

(a) 5048 (b) 5050 (c) 5060 (d) 5064

PART III. All answers should be in simplest form. Each correct answer is worth six points.

1. In the right triangle ABC where $m\angle B = 90°$, $BC : AB = 1 : 2$. Construct the median BD and let point E be on BD such that $CE \perp BD$. Determine $BE : ED$.



2. Suppose a function $f : \mathbb{R} \to \mathbb{R}$ satisfies the following conditions:

   (a) $f(4xy) = 2y\,[f(x+y) + f(x-y)]$, for all $x, y \in \mathbb{R}$

   (b) $f(5) = 3$

   Find the value of $f(2015)$.

3. Let $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ be a polynomial with real coefficients and satisfying the property $P(n) = 10n$ for $n = 1, 2, 3, 4,$ and 5. Find the value of $a + b + c + d + e$.

4. Let $N = \{0, 1, 2, 3, \ldots\}$. Find the cardinality of the set

$$\{(a, b, c, d, e) \in N^5 : 0 \leq a + b \leq 2, 0 \leq a + b + c + d + e \leq 4\}.$$

5. How many terms are there when the expression of $(x+y+z)^{2015} + (x-y-z)^{2015}$ is expanded and simplified?

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PART I. Choose the best answer. Each correct answer is worth two points.

1. B
2. C
3. C
4. B
5. A
6. B
7. D
8. B
9. D
10. B
11. B
12. D
13. B
14. B
15. B

PART II. Choose the best answer. Each correct answer is worth three points.

1. D
2. D
3. B
4. B
5. B
6. B
7. C
8. C
9. B
10. D

PART III. All answers should be in simplest form. Each correct answer is worth six points.

1. $2 : 3$
2. 1209
3. 9
4. 105
5. 1016064