



13th Philippine Mathematical Olympiad

Qualifying Stage

23 October 2010

Part I. Each correct answer is worth two points.

1. What is the sum of the roots of $x^2 - 2009x - 2010 = 0$?
(a) 2010 (b) 2009 (c) 2011 (d) -2010
2. Find the value of $2\sqrt{2\sqrt{2\sqrt{2}\cdots}}$.
(a) 2 (b) $\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$
3. If $2^{2^x} = 4^3$, what is x ?
(a) $\log_2 6$ (b) $\log_4 6$ (c) $\log_6 2$ (d) $\log_6 4$
4. For what values of a does the system

$$\begin{cases} x^2 - y^2 & = 0 \\ (x - a)^2 + y^2 & = 0 \end{cases}$$

have a unique solution?

- (a) $a = -1$ (b) $a = 0$ (c) $a = 1$ (d) $a = 2$
5. If $x + y = 4$ and $x^2 + y^2 = 10$, what is the value of $x^4 + y^4$?
(a) 84 (b) 100 (c) 68 (d) 82
6. Let f be a function defined on the set of integers such that $f(1) = 5$ and $f(x + 1) = 2f(x) + 1$ for all integers x . What is the value of $f(7) - f(0)$?
(a) 380 (b) 189 (c) 191 (d) 381
7. There are k zeros at the end of $34! = 34 \cdot 33 \cdot 32 \cdots 4 \cdot 3 \cdot 2 \cdot 1$. What is the value of k ?
(a) 7 (b) 4 (c) 6 (d) 5

8. Find the sum $\cos 1^\circ + \cos 3^\circ + \cos 5^\circ + \cdots + \cos 177^\circ + \cos 179^\circ$.
- (a) $\frac{\sqrt{2}}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$
9. If $\frac{18x + 7y}{12y + 5x} = \frac{2}{3}$, what is the value $\frac{x}{y}$?
- (a) $\frac{57}{46}$ (b) $\frac{44}{3}$ (c) $\frac{46}{57}$ (d) $\frac{3}{44}$
10. A 4 by 6 inch paper is folded so that its upper right corner touches the midpoint of an opposite side and such that the fold obtained is the longer one. Find the length of the fold.
- (a) $2\sqrt{13}$ in (b) 5 in (c) $\sqrt{65}$ in (d) $5\frac{5}{24}$ in
11. If $a - b + c = 1$, $b - 2c = 0$, $2a + c = 5$, what is the sum $a + b + c$?
- (a) 3 (b) 4 (c) 5 (d) 0
12. A triangle is formed inside a circle by connecting the center C to two points A and B on the circle. If $\angle ACB = 30^\circ$, what is the ratio of the areas of the circle to the triangle?
- (a) $6\pi : 1$ (b) $9 : 1$ (c) $4\pi : 1$ (d) $9\pi : 2$
13. A ball rebounds each time to a height which is half that of the previous one. If the total distance traveled before coming to rest is 72 meters, from how high was the ball dropped?
- (a) 24 meters (b) 18 meters (c) 36 meters (d) 12 meters
14. Let f be the function defined by $f(x) = \frac{\pi^x + \pi^{-x}}{\pi^x - \pi^{-x}}$. Find $f(2p)$ if $f(p) = 2$.
- (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) 4
15. If $x > 0$, find the solution set of $\log x \geq \log 2 + \log(x - 1)$.
- (a) $(1, 2]$ (b) $(-\infty, 2]$ (c) $(0, 1]$ (d) $(\sqrt{2}, 1]$

Part II. Each correct answer is worth three points.

1. Solve for (x, y) in the system $(e^x + 2)^2 - y = 3$, $4(e^x + 2) - y = -1$.

- (a) $(\sqrt{2}, 3)$ (c) $(\ln \sqrt{2}, 3)$
(b) $(\ln 2\sqrt{2}, 9 + 8\sqrt{2})$ (d) $(\ln \sqrt{2}, 2 + 4\sqrt{2})$

2. Mica has six differently colored crayons. She can use one or more colors in her painting. What is the likelihood that she will use only her favorite color?

- (a) $\frac{1}{24}$ (b) $\frac{1}{48}$ (c) $\frac{1}{81}$ (d) $\frac{1}{63}$

3. If $b_1 = \frac{1}{3}$ and $b_{n+1} = \frac{1 - b_n}{1 + b_n}$, for $n \geq 2$, find $b_{2010} - b_{2009}$.

- (a) $\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

4. Let

$$x = 1 - \frac{1}{2 - \frac{1}{1 - \frac{1}{2 - \frac{1}{1 - \dots}}}}$$

Find $(2x - 1)^2$.

- (a) 4 (b) -4 (c) 8 (d) -8

5. $\cos 15^\circ$ is equal to

- (a) $\sqrt{\frac{2 - \sqrt{3}}{2}}$ (b) $\sqrt{\frac{2 - \sqrt{3}}{4}}$ (c) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (d) $\frac{\sqrt{6} + \sqrt{2}}{4}$

6. Solve for x in the equation $\frac{(\log_5 x)^2 - 4}{(\log_5 x)^2 + \log_5 x^4 + 4} + 2\log_5 x = -1$.

- (a) $x = 1$ (b) $x = -1$ (c) $x = 2$ (d) $x = 3$

7. A line with y -intercept 5 and positive slope is drawn such that this line intersects $x^2 + y^2 = 9$. What is the least slope of such a line?

- (a) $\frac{1}{3}$ (b) 1 (c) $\frac{5}{6}$ (d) $\frac{7}{6}$

8. A metal bar bent into a square is to be painted. How many distinct ways can one color the metal bar using four distinct colors on the edges using red, white, blue, and yellow.

- (a) 8 (b) 24 (c) 3 (d) 4

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Answer Key

I. (Two points each)

1. b
2. c
3. a
4. b
5. d
6. d
7. a
8. c
9. d
10. d
11. c
12. c
13. a
14. c
15. a

II. (Three points each)

1. b
2. d
3. c
4. This question was discarded since the given continued fraction is divergent. If the question of convergence is not taken into account, the answer should be -1.
5. d
6. a
7. This question was discarded because the correct answer was not among the choices. The slope is $\frac{4}{3}$.
8. c
9. a
10. This question was discarded because the correct answer was not among the choices. There are 30 such arrangements possible.

III. (Six points each)

1. c
2. b
3. d
4. b
5. d