# 15TH PHILIPPINE MATHEMATICAL OLYMPIAD

2013

# ABOUT THE PMO

First held in 1984, the PMO was created as a venue for high school students with interest and talent in mathematics to come together in the spirit of friendly competition and sportsmanship. Its aims are: (1) to awaken greater interest in and promote the appreciation of mathematics among students and teachers; (2) to identify mathematically-gifted students and motivate them towards the development of their mathematical skills; (3) to provide a vehicle for the professional growth of teachers; and (4) to encourage the involvement of both public and private sectors in the promotion and development of mathematics education in the Philippines.

The PMO is the first part of the selection process leading to participation in the International Mathematical Olympiad (IMO). It is followed by the Mathematical Olympiad Summer Camp (MOSC), a five-phase program for the twenty national finalists of PMO. The four selection tests given during the second phase of MOSC determine the tentative Philippine Team to the IMO. The final team is determined after the third phase of MOSC.

The PMO this year is the fifteenth since 1984. Almost three thousand five hundred (3500) high school students from all over the country took the qualifying examination, out of these, two hundred eight (208) students made it to the Area Stage. Now, in the National Stage, the number is down to twenty and these twenty students will compete for the top three positions and hopefully move on to represent the country in the 54th IMO, which will be held in Santa Marta, Colombia on July 18-28, 2013.

# MESSAGE FROM DOST-SEI

Greetings of peace!

With the ushering of a new year, we welcome 2013 with the much awaited commencement of the Final Round of the country's most prestigious mathematics competition, the Philippine Mathematical Olympiad (PMO).

The nation's oldest competition has continuously proven its effectiveness in discovering and advancing exemplary talent in mathematics. Since 2008, the PMO has enabled the country to carry on its medal haul in the International Mathematics Olympiad, the most prestigious mathematics competition in the world. Indeed, the high degree of competition in the PMO brings out the paramount potential of our students in mathematics.

As medals from international math and science competitions keep on pouring for the Philippines, expectations soar even higher. We believe that the Filipino students have the adequate knowledge and analytic skills that are at par with their international counterparts. And with the implementation of the K-12 program, it is expected that the Philippine contingent will soon rise above its contenders and finally dominate in the international scene.

The Science Education Institute upholds its commitment in supporting the Mathematics Society of the Philippines in nurturing the Filipino youth who are gifted in mathematics. We are positive that this year's PMO will harvest a new crop of outstanding change-makers in the S&T landscape.

We look forward to an exciting PMO and we wish all the contestants the best.

FILMA G. BRAWNER, Ph. D.
Director, Science Education Institute
Department of Science and Technology

# MESSAGE FROM DEP-ED

I salute the country's elite, young mathematical talents who will one day soon make our country proud in the international math circle.

I understand that the organizers, the trainers, coaches and parents are all doing their best to ensure that our math wizards are getting the best training an preparation possible to make them emerge as winners in the world-calibre competition. To you, who serve as mentors to our competitions, I send my admiration.

To our young math talents, may the training you get and the possible accolades you will continue to reap be reverted to our country by taking active roles in science and technology initiatives in the future.

Forward you go!

BR. ARMIN A. LUSTRO FSC

Secretary

Department of Education

# MESSAGE FROM MSP

For close to four decades now, the MSP has been involved in organizing mathematics competitions, starting with the Metro Manila Math Competition in 1977. MSP is proud to organize the Philippine Mathematical Olympiad, the toughest and most prestigious math competition in the country. The PMO was first held in 1984 but the first National Level Competitions of the PMO was held in SY1986-1987. We are grateful to the Department of Science and Technology-Science Education Institute (DOST-SEI) for partnering with us in organizing this activity.

The aim of the Philippine Mathematical Olympiad is to identify and reward excellence in Mathematics. We hope to discover, motivate and nurture talents and hopefully steer them to careers in Science and Mathematics in the future. The participants have displayed good Filipino values such as determination, hard work and optimism. Congratulations to the winners and all the participants of the 15th PMO!

Congratulations to the coaches and the school administrators. We hope competitions such as this provide constant stimulus for improving teaching and the curriculum in your schools. The MSP believes that competitions are important in promoting stronger educational culture.

In behalf of the MSP, I wish to thank the sponsors, schools and other organizations, institutions and individuals for their continued support and commitment to the PMO. Thank you and congratulations to Dr. Jose Ernie Lope and his team for the successful organization of the 15th PMO.

Jumela Sarmiento

JUMELA F. SARMIENTO, Ph.D.
President
Mathematical Society of the Philippines

# MESSAGE FROM FUSE

Congratulations to the Philippine Mathematical Olympiad!

In any endeavor, excellence does not happen by chance neither by talent alone. Excellence comes about and results from effort, determination and persistence.

In this regard, I share the views of the Philippine Mathematical Olympiad. We both work with deep enthusiasm and interest towards building national excellence and talents in Mathematics through national competitions which identify talented and gifted children in Mathematics.

I congratulate the Philippine Mathematical Olympiad in its laudable effort at identifying, building up and sustaining the interest and talents of students in the Philippines.

LUCIO C. TAN
Vice-Chairman
FUSE

# MESSAGE FROM C&E

I write this message as I listen to news about the Philippine Azkals Football team preparing for a fight with Spanish players coming very soon to the Philippines. The celebrity status achieved by the Azkals members is an indication of how promising this re-discovered sports is to Filipinos. Here is finally a sports where people, regardless of height or country of origin can excel.

I would like to think of Mathematics as the football of high school subjects and the PMO as the Azkals of international scholastic competitions.

Mathematics as a subject does level the playing field and the success of Filipino students in this subject when competing abroad is testimony to how the Philippines can keep earning another reputation for being home to world-class Math champions.

In keeping with my personal belief that we indeed have the best Math students this side of the planet, rest assured that C&E Publishing, Inc. will always be behind the Philippine Math Olympiad in the Organization's noble quest to produce the brightest of young mathematicians.

Congratulations to all the qualifiers to the National Level. Congratulations to the members and officers of the Philippine Math Olympiad for once again staging and now having the 15th Philippine Mathematical Olympiad.

May your effort keep on exponentially multiplying into the highest Mersenne prime possible. Mabuhay!

EMYL EUGENIO
VP-Sales and Marketing Division
C&E Publishing, Inc.

# THE PMO FINALISTS

**Clyde Wesly Si Ang**
Chiang Kai Shek College
Frederick Buiza

**John Angel Libranda Aranas**
Makati Science High School
Mark Vidallo

**Deany Hendrick Cheng**
Grace Christian College
Josephine Sy-Tan

**Jervis Ong Chua**
Philippine Cultural College
Reynaldo Sy, Jr.

**John Thomas Yu Chuatak**
St. Stephen's High School
Tom Ng Chu

**Kyle Patrick Flor Dulay**
Philippine Science High School - Main
Fortunato Tacuboy III

**Brendon Di Go**
British School Manila

**Ma. Czarina Angela Sio Lao**
St. Jude Catholic School
Manuel Tanpoco

**Tiong Soon Kelsey Lim**
Grace Christian College
Josephine Sy-Tan

**Lu Christian Sy Ong**
Grace Christian College
Josephine Sy-Tan

**Lorenzo Gabriel Del Rosario Quiogue**
Ateneo de Manila High School
Ryan M. Bruce

**Reine Jiana Mendoza Reynoso**
Philippine Science High School - Main
Jose Manresa Enrico Español

**Adrian Reginald Chua Sy**
St. Jude Catholic School
Manuel Tanpoco

**Andrew Kua Sy**
Xavier School
Linda May Hernandez

**Benson Michael Tan Tan**
Jubilee Christian Academy
Enrique Sabacan

**Jason Allan Tan Tan**
Jubilee Christian Academy
Enrique Sabacan

**Matthew Sy Tan**
St. Jude Catholic School
Manuel Tanpoco

**Mikaela Angelma Chan Uy**
St. Jude Catholic School
Manuel Tanpoco

**Farrell Eldrian So Wu**
MGC New Life Christian Academy
Neshie Joyce Guntiñas

**Justin Edric Go Yturzaeta**
Jubilee Chrisitian Academy
Enrique Sabacan

# THE PMO TEAM

**Director**

Jose Ernie Lope

**Assistant Directors**

Richard Lemence

Dranreb Earl Juanico

Yvette Lim

**Test Development Committee**

Job Nable

Clark Go

Mark Tolentino

Timothy Teng

Flor Francisco

**Treasurer**

Victoria May Paguio

**Logistics and Operations Committee**

May Anne Tirado

Jared Guissmo Asuncion

Paul Reine Kennett De La Rosa

Guadella Ruiz

Charles Repizo

**National Stage Preparations**

Rechel Arcilla

Kristine Carpio

Frumencio Co

Isagani Jos

Ederline Nocon

Shirlee Ocampo

Anita Ong

Arturo Pacificador Jr.

Arlene Pascasio

Rigor Ponsones

Jose Tristan Reyes

Leonor Ruivivar

Sonia Tan

Regina Tresvalles

## Regional Coordinators

**Region 1 / CAR**
Prof Wilfredo Alangui

**Region 2**
Mr Crizaldy Binarao

**Region 3**
Dr Jumar Valdez

**Region 4A**
Mr John Mark Lampos

**Region 4B**
Dr Elucila Sespene

**Region 5**
Ms Cresencia Laguerta

**Region 6**
Prof Filame Joy Catinan

**Region 7**
Dr Lorna Almocera

**Region 8**
Mr Jonas Villas

**Region 9**
Dr Rochelleo Mariano

**Region 10**
Dr Jocelyn Vilela

**Region 11**
Dr Eveyth Deligero

**Region 12 / ARMM**
Dr Jonald Pimentel

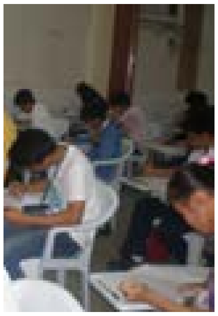
**Region 13**
Dr Thelma Montero-Galliguez

**NCR**
Dr Recto Rex Calingasan

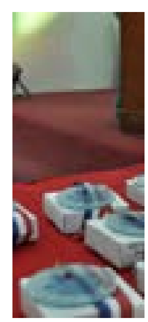
## Schedule

| | |
|---|---|
| 0730AM - 0830AM | **Registration** |
| 0900AM - 1200NN | **Phase I: Written Phase** |
| 1200NN - 0200PM | **Lunch Break** |
| 0200PM - 0500PM | **Phase II: Oral Phase** |
| | **National Anthem** |
| | **Welcoming Remarks** |
| | **Awarding of Certificates** |
| | **Oral Competition** |
| | **IMO Summer Camp Briefing** |
| 0630PM - 0830PM | **Dinner & Awarding Ceremonies** |

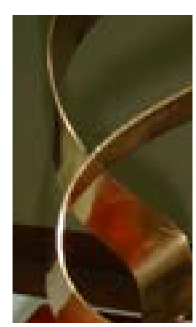
# PMO: THROUGH THE YEARS











**PART I.** Each correct answer is worth two points.

1. Find the sum of

$$\frac{1}{2\times 5}+\frac{1}{5\times 8}+\frac{1}{8\times 11}+\cdots+\frac{1}{2009\times 2012}.$$

(a) $\frac{335}{2012}$ (b) $\frac{545}{2012}$ (c) $\frac{865}{2012}$ (d) $\frac{1005}{2012}$

2. Find the last two digits of $0!+5!+10!+15!+\cdots+100!$.

(a) 00 (b) 11 (c) 21 (d) 01

3. Consider the system

$$xy=10^a, yz=10^b, xz=10^c.$$

What is $\log x+\log y+\log z$?

(a) $\frac{abc}{2}$ (b) $\frac{a+b+c}{2}$ (c) $a+b+c$ (d) $abc$

4. A polyhedron has 30 faces and 62 edges. How many vertices does the polyhedron have?

(a) 61 (b) 34 (c) 46 (d) 77

5. Which of the following quadratic expressions in $x$ have roots $\frac{g}{h}$ and $-\frac{h}{g}$?

(a) $g^2h^2x^2-\frac{g^2}{h^2}$
(b) $hgx^2+(g^2-h^2)x-hg$
(c) $hgx^2+(h^2-g^2)x+hg$
(d) $hgx^2+(h^2-g^2)x-hg$

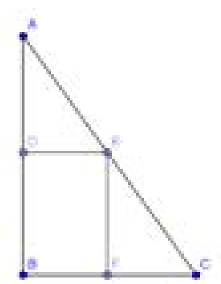
6. If $x^6=64$ and $\left(\frac{2}{x}-\frac{x}{2}\right)^2=b$, then a function $f$ that satisfies $f(b+1)=0$ is

(a) $f(x)=1-2^{x-1}$
(b) $f(x)=2^{x-1}$
(c) $f(x)=x^2+x$
(d) $f(x)=2x-1$

7. Sixty men working on a construction job have done 1/3 of the work in 18 days. The project is behind schedule and must be accomplished in the next twelve days. How many more workers need to be hired?

   (a) 60 (b) 180 (c) 120 (d) 240

8. The vertices $D$, $E$ and $F$ of the rectangle are midpoints of the sides of $\triangle ABC$. If the area of $\triangle ABC$ is 48, find the area of the rectangle.



   (a) 12 (b) 24 (c) 6 (d) $12\sqrt{2}$

9. Determine the number of factors of $5^x + 2 \cdot 5^{x+1}$.

   (a) $x$ (b) $x+1$ (c) $2x$ (d) $2x+2$

10. How many solutions has $\sin 2\theta - \cos 2\theta = \sqrt{6}/2$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$?

   (a) 1 (b) 2 (c) 3 (d) 4

11. If $2\sin(3x) = a\cos(3x + c)$, find all values of $ac$. In the choices below, $k$ runs through all integers.

   (a) $-\frac{\pi}{2}$
   (b) $2k\pi$
   (c) $-\pi$
   (d) $(4k-1)\pi$

12. If $x$ satisfies $\dfrac{\log_2 x}{\log_2 2x - \log_8 2} = 3$, the value of $1 + x + x^2 + x^3 + x^4 + \cdots$ is

   (a) 1
   (b) 2
   (c) $\frac{1}{2}$
   (d) the value does not exist

13. Find the least common multiple of 15! and $2^3 3^9 5^4 7^1$.

(a) $2^3 3^6 5^3 7^1 11^1 13^1$
(b) $2^3 3^6 5^3 7^1$
(c) $2^{11} 3^9 5^4 7^2 11^1 13^1$
(d) $2^{11} 3^9 5^4 7^2$

14. If $(a, b)$ is the solution of the system $\sqrt{x+y}+\sqrt{x-y}=4,\ x^2-y^2=9$, then $\dfrac{ab}{a+b}$ has value

(a) $\dfrac{10}{9}$ (b) $\dfrac{8}{3}$ (c) 10 (d) $\dfrac{20}{9}$

15. Find the value of $\sin\theta$ if the terminal side of $\theta$ lies on the line $5y-3x=0$ and $\theta$ is in the first quadrant.

(a) $\dfrac{3}{\sqrt{34}}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{4}{\sqrt{34}}$

**PART II.** Each correct answer is worth three points.

1. Find the value of $\log_2[2^3 4^4 8^5 \cdots (2^{20})^{22}]$.

(a) 3290 (b) 3500 (c) 3710 (d) 4172

2. Let $2=\log_b x$. Find all values of $\dfrac{x+1}{x}$ as $b$ ranges over all positive real numbers.

(a) $(0,+\infty)$
(b) $(1,+\infty)$
(c) $(0,1)$
(d) all real numbers

3. Solve the inequality $\dfrac{1}{3^x}\left(\dfrac{1}{3^x}-2\right)<15$.

(a) $\left(-\dfrac{\log 5}{\log 3},+\infty\right)$
(b) $\left(-\infty,\dfrac{\log 5}{\log 3}\right)$
(c) $\left(\dfrac{\log 3}{\log 5},1\right)$
(d) $(\log 3,\log 5)$

4. Find the solution set of the inequality $\left(\frac{\pi}{2}\right)^{(x-1)^2} \leq \left(\frac{2}{\pi}\right)^{x^2-5x-5}$.

   (a) $[-1, 4]$
   (b) $(-\infty, -1] \cup (4, +\infty)$
   (c) $[-1/2, 4]$
   (d) $(-\infty, -1/2] \cup [4, +\infty)$

5. The equation $x^2 - bx + c = 0$ has two roots $p$ and $q$. If the product $pq$ is to be maximum, what value of $b$ will make $b + c$ minimum?

   (a) $-c$
   (b) $-2$
   (c) $\frac{1}{2}$
   (d) $2c$

6. Find the range of the function $f(x) = \dfrac{6}{5\sqrt{x^2 - 10x + 29} - 2}$.

   (a) $[-1/2, 3/4]$
   (b) $(0, 3/4]$
   (c) $(1/2, 4/3]$
   (d) $[-1/2, 0) \cup (0, 4/3]$

7. A fair die is thrown three times. What is the probability that the largest outcome of the three throws is a 3?

   (a) 1/36
   (b) 19/216
   (c) 25/36
   (d) 1/8

8. If $f$ is a real-valued function, defined for all nonzero real numbers, such that $f(a) + \dfrac{1}{f(b)} = f\left(\dfrac{1}{a}\right) + f(b)$, find all possible values of $f(1) - f(-1)$.

   (a) $\{-2, 2\}$
   (b) $\{0, -1, 1\}$
   (c) $\{1, 2\}$
   (d) $\{0, -2, 2\}$

9. Which is a set of factors of $(r-s)^3 + (s-t)^3 + (t-r)^3$?

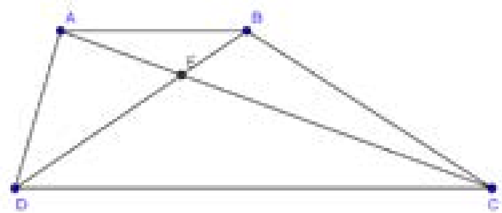
   (a) $\{r-s, s-t, t-r\}$
   (b) $\{3r-3s, s+t, t+r\}$
   (c) $\{r-s, s-t, t-2rt+r\}$
   (d) $\{r^2-s^2, s-t, t-r\}$

10. If $36 - 4\sqrt{2} - 6\sqrt{3} + 12\sqrt{6} = (a\sqrt{2} + b\sqrt{3} + c)^2$, find the value of $a^2 + b^2 + c^2$.

   (a) 12
   (b) 5
   (c) 14
   (d) 6

**PART III.** Each correct answer is worth six points.

1. $ABCD$ is a trapezoid with $AB||CD$, $AB = 6$ and $CD = 15$. If the area of $\triangle AED = 30$, what is the area of $\triangle AEB$?



(a) 20 (b) 40/7 (c) 12 (d) 8/3

2. Find the maximum value of $y = (7-x)^4(2+x)^5$ when $x$ lies strictly between $-2$ and 7.

(a) $7^4 2^5$ (c) $(2.5)^9$
(b) $(4.5)^4(2.5)^5$ (d) $(4.5)^9$

3. Find all possible values of $\dfrac{2\cdot 3^{-x}-1}{3^{-x}-2}$, as $x$ runs through all real numbers.

(a) $(-\infty, 1/2)\cup(2,+\infty)$ (c) $[2,+\infty]$
(b) $(1/2, 2)$ (d) $(0,+\infty)$

4. In how many ways can one select five books from a row of twelve books so that no two adjacent books are chosen?

(a) 34 (b) 78 (c) 42 (d) 56

5. Find the range of

$$f(x) = \frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)}$$

where $a, b, c$ are distinct real numbers.

(a) all real numbers (c) $[-a-b-c,+\infty)$
(b) $\{1\}$ (d) $\{a+b+c\}$

**Part I.** No solution is needed. All answers must be in simplest form. Each correct answer is worth three points.

1. Find all complex numbers $z$ such that $\dfrac{z^4+1}{z^4-1} = \dfrac{i}{\sqrt{3}}$.
2. Find the remainder if $(2001)^{2012}$ is divided by $10^6$.
3. If $z^3 - 1 = 0$ and $z \neq 1$, find the value of $z + \dfrac{1}{z} + 4$.
4. Find the equation of the line that contains the point $(1,0)$, that is of least positive slope, and that does not intersect the curve $4x^2 - y^2 - 8x = 12$.
5. Consider a function $f(x) = ax^2+bx+c, a > 0$ with two distinct roots a distance $p$ apart. By how much, in terms of $a, b, c$ should the function be translated downwards so that the distance between the roots becomes $2p$?
6. Find the equation of a circle, in the form $(x-h)^2 + (y-k)^2 = r^2$, inscribed in a triangle whose vertex are located at the points $(-2,1), (2,5), (5,2)$.
7. Define $f(x) = \dfrac{a^x}{a^x + \sqrt{a}}$ for any $a > 0$. Evaluate $\displaystyle\sum_{i=1}^{2012} f\left(\frac{i}{2013}\right)$.
8. Let $3x, 4y, 5z$ form a geometric sequence while $\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}$ form an arithmetic sequence. Find the value of $\dfrac{x}{z} + \dfrac{z}{x}$.
9. Consider an acute triangle with angles $\alpha, \beta, \gamma$ opposite the sides $a, b, c$, respectively. If $\sin\alpha = \dfrac{3}{5}$ and $\cos\beta = \dfrac{5}{13}$, evaluate $\dfrac{a^2+b^2-c^2}{ab}$.
10. A change from Cartesian to polar coordinates involves the following transformation: $x = r\cos\theta$ and $y = r\sin\theta$. For a circle with polar equation $r = \dbinom{m}{n}\cos\theta$, where $1 \leq n \leq m \leq 6$, how many distinct combinations of $m$ and $n$ will this equation represent a circle of radius greater than or equal to 5?
11. Let $f$ be a polynomial function that satisfies $f(x-5) = -3x^2 + 45x - 108$. Find the roots of $f(x)$.
12. Six boy-girl pairs are to be formed from a group of six boys and six girls. In how many ways can this be done?

13. From the $xy$-plane, select five distinct points that have integer coordinates. Find the probability that there is a pair of points among the five whose midpoint has integer coordinates.

14. Given that $\tan\alpha + \cot\alpha = 4$, find $\sqrt{\sec^2\alpha + \csc^2\alpha - \frac{1}{2}\sec\alpha\csc\alpha}$.

15. There are 100 people in a room. 60 of them claim to be good at math, but only 50 are actually good at math. If 30 of them correctly deny that they are good at math, how many people are good at math but refuse to admit it?

16. Find the value of $p$, where,

$$p = \frac{16^2-4}{18\times 13} \times \frac{16^2-9}{19\times 12} \times \frac{16^2-16}{20\times 11} \times \cdots \times \frac{16^2-64}{24\times 7}.$$

17. The number $x$ is chosen randomly from the interval $(0,1]$. Define $y = \lceil \log_4 x \rceil$. Find the sum of the lengths of all subintervals of $(0,1]$ for which $y$ is odd. For any real number $a$, $\lceil a \rceil$ is defined as the smallest integer not less than $a$.

18. Find the value/s of $k$ so that the inequality $k(x^2+6x-k)(x^2+x-12) > 0$ has solution set $(-4,3)$.

19. A sequence of numbers is defined using the relation

$$a_n = -a_{n-1} + 6a_{n-2}$$

where $a_1 = 2, a_2 = 1$. Find $a_{100} + 3a_{99}$.

20. Define the following operation for real numbers: $a \star b = ab + a + b$. If $x \star y = 11$, $y \star z = -4$, and $x \star z = -5$. What is the difference between the maximum and minimum elements of the solution set $\{x, y, z\}$?

**Part II.** Show the solution to each item. Each complete and correct solution is worth ten points.

1. If $x + y + xy = 1$, where $x, y$ are nonzero real numbers, find the value of

$$xy + \frac{1}{xy} - \frac{y}{x} - \frac{x}{y}.$$

2. The quartic (4th-degree) polynomial $P(x)$ satisfies $P(1) = 0$ and attains its maximum value of 3 at both $x = 2$ and $x = 3$. Compute $P(5)$.

3. Let $v(X)$ be the sum of elements of a nonempty finite set $X$, where $X$ is a set of numbers. Calculate the sum of all numbers $v(X)$ where $X$ ranges over all nonempty subsets of the set $\{1, 2, 3, ..., 16\}$.

# ANSWER KEY

## Qualifying Round

I.
1. A
2. C
3. B
4. B
5. D
6. A
7. C
8. B
9. D
10. D
11. D
12. B
13. C
14. D
15. A

II.
1. A
2. B
3. A
4. C
5. B
6. B
7. B
8. D
9. A
10. C

III.
1. C
2. D
3. A
4. D
5. B

## Area Stage

I.
1. $\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{\sqrt{3}}{2}+\frac{1}{2}i,$ $-\frac{1}{2}-\frac{\sqrt{3}}{2}i,\ \frac{\sqrt{3}}{2}-\frac{1}{2}i$
2. 24001
3. 3
4. $y=2x-2$
5. $\dfrac{3b^2}{4a}-3c$
6. $(x-2)^2+(y-3)^2=2$
7. 1006
8. $\dfrac{34}{15}$
9. $\dfrac{32}{65}$
10. 5
11. 7, -2
12. 6!
13. 1
14. $\sqrt{14}$
15. 10
16. 2
17. 1/5
18. $(-\infty,-9]$
19. $7\times 2^{98}$
20. 5

II.
1. Observe that

$$\begin{aligned} xy+\frac{1}{xy}-\frac{y}{x}-\frac{x}{y} &= \frac{(xy)^2+1-x^2-y^2}{xy} \\ &= \frac{(x^2-1)(y^2-1)}{xy} \\ &= \frac{(x+1)(y+1)(x-1)(y-1)}{xy} \\ &= (xy+x+y+1)\frac{(xy-x-y+1)}{xy}. \end{aligned}$$

Since, $xy + x + y = 1$, the first term will equal to 2. Moreover, dividing both sides of the equation $xy + x + y = 1$ by $xy$, we obtain

$$1 + \frac{1}{y} + \frac{1}{x} = \frac{1}{xy},$$

which is equivalent to

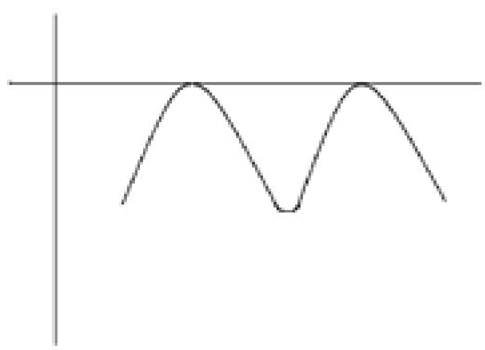
$$1 = \frac{1}{xy} - \frac{1}{y} - \frac{1}{x}.$$

Hence, $(xy + x + y + 1)\dfrac{(xy - x - y + 1)}{xy} = 2 \cdot 2 = \boxed{4}$.

2. Consider the polynomial $Q(x) = P(x) - 3$. Then $Q(x)$ has zeros at and maximum value 0 at $x = 2, 3$. These conditions imply that $Q(x)$ has the form

$$Q(x) = A(x-2)^2(x-3)^2.$$

That is, its graph looks like



because the values of $Q(x)$ should grow larger and larger through negative values as the variable $x$ goes to larger and larger values of both signs and the fact that the number of turning points should not exceed $4 - 1 = 3$ but should be more than 2 (given by the maximum points). Thus, $Q(1) = -3$ imply $a = -\dfrac{3}{4}$. Finally, $Q(5) = P(5) - 3 = -27$ and $P(5) = \boxed{-24}$.

3. The answer is $2^{15} \cdot 8 \cdot 17$

We note that each $k \in \{1, 2, 3, ..., 16\}$ belongs to $2^{15}$ subsets of $\{1, 2, 3, ..., 16\}$. We reason as follows: we can assign 0 or 1 to $k$ according to whether it is not or in a subset of {1, 2, 3,..., 16}. As there are 2 choices for a fixed $k$, $k$ belongs to half of the total number of subsets, which is $2^{16}$. Hence the sum is

$$\sum v(X) = 2^{15}(1 + 2 + 3 + \cdots + 16) = 2^{15} \cdot 8 \cdot 17 = \boxed{4456448}$$

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