
2012
PHILIPPINE
MATHEMATICAL
OLYMPIAD
MCMLXXXIV
PMO

The Contest

First held in 1984, the PMO was created as a venue for high school students with interest and talent in mathematics to come together in the spirit of friendly competition and sportsmanship. Its aims are: (1) to awaken greater interest in and promote the appreciation of mathematics among students and teachers; (2) to identify mathematically-gifted students and motivate them towards the development of their mathematical skills; (3) to provide a vehicle for the professional growth of teachers; and (4) to encourage the involvement of both public and private sectors in the promotion and development of mathematics education in the Philippines

The PMO is the first part of the selection process leading to participation in the International Mathematical Olympiad (IMO). It is followed by the Mathematical Olympiad Summer Camp (MOSC), a five-phase program for the twenty national finalists of PMO. The four selection tests given during the second phase of MOSC determine the tentative Philippine Team to the IMO. The final team is determined after the third phase of MOSC.

The PMO this year is the fourteenth since 1984. Three thousand five hundred ninety-six (3596) high school students from all over the country took the qualifying examination, out of these, two hundred three (203) students made it to the Area Stage. Now, in the National Stage, the number is down to twenty and these twenty students will compete for the top three positions and hopefully move on to represent the country in the 53rd IMO, which will be held in Mar del Plata, Argentina on July 4-16, 2010.

Message from DOST-SEI

Congratulations to all participants and oranizers of the 14th Philippine Mathematical Olympiad!

The Science Education Institute of the Department of Science and Technology, the Philippines' science and technology human resource development agency, with its mandate of creating a critical mass of S&T professionals in the country supports the 14th PMO in its search for the top math wizards who will be the country's representatives to the 53rd International Math Olympiad to be held in Mar de Plata, Argentina.

We congratulate as well the Mathematical Society of the Philippines for its untirin efforts in searching for the buddin minds in mathematics who we hope will join us in our dynamic science community in the future. We also thank the MSP for honing the mathematical aptitudes of the teams we send to the IMO, continuing the nation's medal streak for the last three years.

For our part, SEI shall remain in its commitment to provide lampposts to our future scientists, engineers, and science teachers through incentives such as S&T scholarships and innovative delivery systems in an effort to create a culture of science in the Philippines, and eventually develop a pool of highly skilled S&T workers.

The greater task at hand is to translate the achievements which our mathematics wizards have attained and enable these young mathematicians to continue their pursuit of excellence as they take more challenging roles as scientists and engineers of this country. We hope that all our PMO participants will take this challenge and help bolster the Philippines' economic development through research and development.

Thank you and mabuhay tayong lahat.

Filma G. Brawner
Director
DOST-SEI

Message from Dep-Ed

I wish to convey cordial greetings to the officers and members of the Matheatical Society of the Philippines (MSP) as they hold the 14th Philippine Mathematical Olympiad, National Finals on January 28 this year.

The Mathematical Society of the Philippines (MSP) is the Department's partner in its bid to develop a more math-oriented studentry. This goal can only be achieved through a group of commited math enthusiasts like the MSP and a more rigid mathematics curriculum.

I am personally glad that the MSP has sponsored this Olympiad. The Department demands from the national winners' utilization of their math talents and skills to the fullest.

On this note, I send my very best wishes for the success of this activity.

Br. Armin A. Lustro FSC
Secretary
Department of Education

Message from MSP

The Mathematical Society of the Philippines (MSP) believes that competitions enhance education. MSP is proud to organize the Philippine Mathematical Olympiad, the toughest and most prestigious math competition in the country. The MSP has been at the forefront of the promotion of mathematics education and research in our country for 39 years. For more than three decades now, the MSP has been involved in organizing math competitions. We are grateful to the Department of Science and Technology-Science Education Institute (DOST-SEI) for partnering with us in organizing this activity. The MSP and DOST-SEI both believe that competitions are important in developing our educational system. Math competitions foster greater peer-learning environments and promote stronger educational culture.

The aim of the Philippine Mathematical Olympiad is to identify and reward excellence in Mathematics. We hope to discover and nurture talents and hopefully attract them to careers in Science and Mathematics in the future. The participants have displayed good Filipino values such as determination, hard work and optimism. Congratulations to the winners and all the participants of the 14th PMO!

In behalf of the MSP, I wish to thank the sponsors, schools and other organizations, institutions and individuals for their continued support and commitment to the PMO. Thank you and congratulations to Dr. Jose Ernie Lope and his team for the successful organization of the 14th PMO.

Jumela Sarmiento

Jumela F. Sarmiento, Ph.D.
President
Mathematical Society of the Philippines

Message from FUSE

It is with much honor and pride that I greet the Philippine Mathematical Olympiad (PMO) Management this year.

To all the finalists in the Philippine Mathematical Olympiad, my congratulations!

Mathematics is my most loved subject. To my mind, it is the best way to search for knowledge, particularly in the sciences - where our country is aiming to make progress. Through students and contestants like you, I am sure that there is hope for progress through discoveries and innovations, which by their nature are anchored on mathematical excellence and prowess.

I hope that you, the brilliant participants in the PMO, will continue the good start and great strides you have made in mathematics in order to contribute significantly to our country's well-being.

More power to you and congratulations again on your outstanding achievement!

Lucio C. Tan
Vice Chairman
FUSE

Message from C&E

I write this message as I listen to news about the Philippine Azkals Football team preparing for a fight with Spanish players coming very soon to the Philippines. The celebrity status achieved by the Azkals members is an indication of how promising this re-discovered sports is to Filipinos. Here is finally a sports where people, regardless of height or country of origin can excel.

I would like to think of Mathematics as the football of high school subjects and the PMO as the Azkals of international scholastic competitions.

Mathematics as a subject does level the playing field and the success of Filipino students in this subject when competing abroad is testimony to how the Philippines can keep earning another reputation for being home to world-class Math champions.

In keeping with my personal belief that we indeed have the best Math students this side of the planet, rest assured that C&E Publishing, Inc. will always be behind the Philippine Math Olympiad in the Organization's noble quest to produce the brightest of young mathematicians.

Congratulations to all the qualifiers to the National Level. Congratulations to the members and officers of the Philippine Math Olympiad for once again staging and now having the 14th Philippine Mathematical Olympiad.

May your effort keep on exponentially multiplying into the highest Mersenne prime possible. Mabuhay!

Emyl Eugenio
VP-Sales and Marketing Division
C&E Publishing, Inc.

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Jose Ernie Lope

Assistant Directors
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Aaron James Porlante

Logistics and Operations Committee
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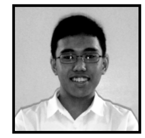
Jasmin-Mae Santos
Jared Asuncion

The Fourteenth PMO Finalists

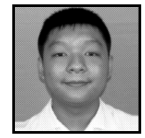
Michael Brodeth
Philippine Science High School - Main
Fortunato A. Tacuboy III

Andrew Joelle Caguntas
Makati Science High School
Mark Anthony Vidallo



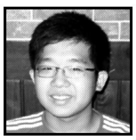
Deany Hendrick Cheng
Grace Christian College
Rosamair Liz Nicol Chua

Sean Timothy Cheng
Grace Christian College
Rosamair Liz Nicol Chua



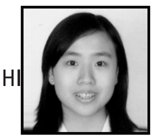
Jervis Chua
Philippine Cultural College - Main
Reynaldo A. Sy Jr.

John Thomas Chuatak
St. Stephen's High School
Tom Chu



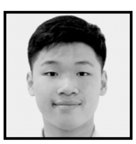
Kenneth Co
Philippine Science High School - Main
Fortunato A. Tacuboy III

Camille Tyrene Dee
Immaculate Conception Academy - Greenhi
Agnes Baranda



Russelle Guadalupe
Valenzuela City Science High School
Marilyn B. Soriano

Marco Vincenzo Karaan
Philippine Science High School - Main
Fortunato A. Tacuboy III

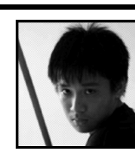


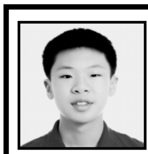
Henry Jefferson Morco
Chiang Kai Shek College
Frederick Buiza

Charles Emerson Ngo
Leyte Progressive High School
Alfredo D. Cruz

Samuel Christian Ong
UNO High School
Kurt Byron C. Ang

Miguel Sebastian Santos
Philippine Science High School - Main
Fortunato A. Tacuboy III





Adrian Reginald Sy
Saint Jude Catholic School
Manuel Tanpoco

Julius Vincent Sy
St. Stephen's High School
Tom Chu

Timothy James Tan
Trinity Christian School
Xerwin Ceballos

Hovve Jan Dominic Teves
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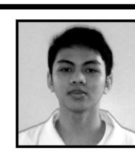
Mark Anthony Vidallo



Mikaela Angelina Uy
Saint Jude Catholic School
Myron Baccay

Justin Edric Yturzaeta
Jubilee Christian Academy
Enrique R. Sabacan

PMO: Through The Years

Schedule of Activities

PHILIPPINE MATHEMATICAL OLYMPIAD

MATHEMATICAL SOCIETY OF THE PHILIPPINES

PHILIPPINE MATHEMATICAL OLYMPIAD
NATIONAL STAGE

NISMED, UP Diliman · January 28, 2012

0730am - 0830am	Registration
0900am - 1200nn	Phase I - Written Phase
1200nn - 0200pm	Lunch Break
0200pm - 0530pm	Phase II - Oral Phase
	National Anthem
	Welcoming Remarks
	Awarding of Certificates
	Oral Competition
	IMO Summer Camp Briefing
0600pm - 0930pm	Dinner and Awarding Ceremonies

Qualifying Round

Part I. Each correct answer is worth two points.

1. Let p and q be the roots of $2x^2 - 5x + 1 = 0$. Find the value of $\log_2 p + \log_2 q$.

   (a) 2 (b) 0 (c) 1 (d) -1

2. Let f be a quadratic function of x. If $2y$ is a root of $f(x - y)$, and $3y$ is a root of $f(x + y)$, what is the product of the roots of $f(x)$?

   (a) $6y^2$ (b) $5y^2$ (c) $4y^2$ (d) $3y^2$

3. If $4 + 12 \cdot 4^x = 16 \cdot 16^x$, what is the value of $2^{2x+4} - 2^{2x}$?

   (a) 120 (b) 60 (c) 30 (d) 15

4. Let $r = \log 50$ and $s = \log 80$. Express $7 \log 20$ in terms of r and s.

   (a) $2r + s$ (b) $2r + 3s$ (c) $r + 2s$ (d) $3r + 2s$

5. Determine the slopes of the lines passing through $\mathrm{P}(3, 0)$ that intersect the parabola with equation $y = 8 - x^2$ at exactly one point.

   (a) $-4, -8$ (b) $-3, -2$ (c) $-5, -7$ (d) $-4, -7$

6. If $\dfrac{b}{x^3} + \dfrac{1}{x^2} + \dfrac{1}{x} + 1 = 0$, what is $x^3 + x^2 + x + a$?

   (a) ab (b) $a + b$ (c) $b - a$ (d) $a - b$

7. How many triangles can be formed if two sides have lengths 15 and 19 and the third side has even length?

   (a) 13 (b) 14 (c) 15 (d) 16

8. Solve for c in the following system of equations:

$$\begin{aligned} 16^{a+b} &= \tfrac{\sqrt{2}}{2} \\ 16^{b+c} &= 4 \\ 16^{a+c} &= 2\sqrt{2} \end{aligned}$$

   (a) 0 (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$

9. Two die are made so that the chances of getting an even sum is twice that of getting an odd sum. What is the probability of getting an odd sum in a single roll of these two die?

   (a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

10. If $\frac{\log x}{\log y} = 500$, what is the value of $\frac{\log(y/x)}{\log y}$?

   (a) -498 (b) -501 (c) -502 (d) -499

11. There are 5 shmacks in 2 shicks, 3 shicks in 5 shures, and 2 shures in 9 shneids. How many shmacks are there in 6 shneids?

    (a) 5 (b) 8 (c) 2 (d) 1

12. Find the sum of the digits of the integer $10^{1001} - 9$.

    (a) 9010 (b) 9001 (c) 9100 (d) 9009

13. What is the constant term in the expansion of $\left(2x^2 + \frac{1}{4x}\right)^6$?

    (a) $\frac{15}{32}$ (b) $\frac{12}{25}$ (c) $\frac{25}{42}$ (d) $\frac{15}{64}$

14. A square with an area of 40m^2 is inscribed in a semicircle. The area of the square that could be inscribed in the circle with the same radius is

    (a) 100m^2 (b) 120m^2 (c) 80m^2 (d) 140m^2

15. What is the units digit of $25^{2010} - 3^{2012}$?

    (a) 8 (b) 6 (c) 2 (d) 4

Part II. Each correct answer is worth three points.

1. Find the area of the region bounded by the graph of $2x^2 - 4x - xy + 2y = 0$ and the x-axis.

    (a) 9 (b) 12 (c) 4 (d) 6

2. Find all negative solutions to the equation $x = \sqrt[3]{20 + 21\sqrt[3]{20 + 21\sqrt[3]{20 + 21x}}}$

    (a) $-1, -2$ (b) $-5, -3$ (c) $-2, -4$ (d) $-4, -1$

3. Find the sum of the largest and smallest possible values of $9\cos^4 x + 12\sin^2 x - 4$.

    (a) 10 (b) 11 (c) 12 (d) 13

4. Exactly one of the following people is lying. Determine the liar.

    Bee said, "Cee is certainly not a liar."

    Cee said, "I know Gee is lying."

    Dee said, "Bee is telling the truth."

    Gee said, "Dee is not telling the truth."

    (a) Bee (b) Cee (c) Dee (d) Gee

5. What is the last digit of $2! + 4! + 6! + \ldots + 2010! + 2012!$?

    (a) 6 (b) 7 (c) 8 (d) 9

6. A point (x, y) is called a lattice point if x and y are integers. How many lattice points are there inside the circle of radius $2\sqrt{2}$ with center at the origin?

   (a) 25 (b) 21 (c) 17 (d) 19

7. Find the least possible value of $|x-1|+|x-3|+|x-5|$.

   (a) 2 (b) 4 (c) 1 (d) 3

8. Let JOHN be a rhombus with JH $=16$ and ON $=12$. Let G and P be points in JN and HN respectively such that JG : GN : NP $=2:2:1$. What is the length GP?

   (a) $\frac{2\sqrt{19}}{5}$ 2.15cm (b) $\frac{2\sqrt{13}}{3}$ (c) $\frac{3\sqrt{17}}{2}$ (d) $\frac{3\sqrt{15}}{2}$

9. If $\sqrt{4+x}+\sqrt{10-x}=6$, find the product $\sqrt{4+x}\sqrt{10-x}$.

   (a) 13 (b) 7 (c) 17 (d) 11

10. The remainders when the polynomial $p(x)$ is divided by $(x+1)$ and $(x-1)$ are 7 and 5, respectively. Find the sum of the coefficients of the odd powers of x.

    (a) -4 (b) 2 (c) -1 (d) 12

Part III. Each correct answer is worth six points.

1. Let $w^3=1$. What is a value of $(1+w-w^2)^3+(1-w+w^2)^3$?

   (a) -16 (b) -21 (c) 18 (d) 15

2. How many positive integer pairs x, y satisfy $\sqrt{x}+\sqrt{y}=\sqrt{600}$?

   (a) 8 (b) 7 (c) 6 (d) 5

3. Evaluate $\cos\frac{\pi}{10}+\cos\frac{2\pi}{10}+\cos\frac{3\pi}{10}+\ldots+\cos\frac{19\pi}{10}$.

   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) -1 (d) 1

4. How many perfect squares divide the number 2!5!6!?

   (a) 18 (b) 15 (c) 20 (d) 25

5. Each of 12 students has a ticket to one of 12 chairs in a row at a theater. How many ways are there to seat the students so that each student sits either in the chair specified on his/her ticket or in one to the left or to the right of the specified chair?

   (a) 233 (b) 225 (c) 187 (d) 252

Area Stage

Part I. Each correct answer is worth three points. No solution is needed.

1. Let $ABCD$ be a rectangle with $AB = 3$ and $BC = 1$. Let E be the midpoint of AB and let F be a point on the extension of DE such that $|CF| = |CD|$. Find the area of $\triangle CDF$.

2. Solve for all real numbers x satisfying
$$x + \sqrt{x-1} + \sqrt{x+1} + \sqrt{x^2-1} = 4.$$

3. The length of a leg of a right triangle is 5 while the length of the altitude to its hypotenuse is 4. Find the length of the other leg.

4. Find all positive values of a for which the equation $x^2 - ax + 1 = 0$ has roots that differ by 1.

5. Let $RALP$ be a trapezoid with $RA||LP$. Let H be the intersection of its diagonals. If the area of $\triangle RAH$ is 9 and the of $\triangle LPH$ is 16, find the area of the trapezoid.

6. The polynomial function $p(x)$ has the form $x^{10} - 4x^9 + \ldots + ax + k$ where $a, k \in \mathbb{R}$. If $p(x)$ has integral zeros, find the minimum possible positive value of k.

7. How many squares are determined by the lines with equations $x = k^2$ and $y = l^2$, where $k, l \in \{0, 1, 2, 3, \ldots, 9\}$?

8. What is the sum of the first 800 terms of $3, 4, 4, 5, 5, 5, 6, 6, 6, 6, \ldots$?

9. Placed on a really long table are 2011 boxes each containing a number of balls. The 1st and the 2nd box together contain 15 balls, the 2nd and the 3rd box together contain 16 balls, the 3rd and the 4th box together contain 17 balls, and so on. If the first and the last box together contain a total of 1023 balls, how many balls are contained in the last box?

10. Evaluate
$$\sqrt[1000]{1000^{1000} + \binom{1000}{1}1000^{998} + \binom{1000}{2}1000^{996} + \cdots + \binom{1000}{999}1000^{-998} + 1000^{-1000}}\,.$$

11. Find all ordered pairs (m, n) of integers such that $4^m - 4^n = 255$.

12. Find all ordered pairs (x, y) satisfying the system
$$\begin{aligned} x^2 + 4y^2 - xy &= 10 \\ 2x - 4y + 3xy &= 11. \end{aligned}$$

13. Find the exact value of $\dfrac{\sqrt{3}}{\sin 20^\circ} - \dfrac{1}{\cos 20^\circ}$.

14. There are two values of r such that $x^4 - x^3 - 18x^2 + 52x + k$ has $x - r$ as a factor. If one of them is $r = 2$, what is the other value of r?

15. For what values of k will the system below have no solution?

$$\begin{aligned} (k-3)x + 2y &= k^2 - 1 \\ x + \left(\frac{k-4}{3}\right) y &= 0 \end{aligned}$$

16. Find all positive integers n such that $n^2 - n + 1$ is a multiple of $5n - 4$.

17. If $x \neq y$ and $\dfrac{x}{y} + x = \dfrac{y}{x} + y$, find the sum $\dfrac{1}{x} + \dfrac{1}{y}$.

18. Let DAN be a triangle whose vertices lie on a circle C. Let AE be the angle bisector of $\angle DAN$ with E on C. If $DA = 2, AN = 1, AE = 2.5$, and AE intersects DN at I, find AI.

19. The length d of a tangent, drawn from a point A to a circle, is $\frac{4}{3}$ of the radius r. What is the shortest distance from A to the circle?

20. If $(x-a)(x-b)(x-c)(x-d) = 9$ is solved by $x = 2$, and a, b, c, and d are distinct integers, find the sum $a + b + c + d$.

Part II. Show your solution for each item. Each item is worth ten points.

1. In rectangle $ABCD$, E and F are chosen on $\overline{AB}$ and $\overline{CD}$, respectively, so that $AEFD$ is a square. If $\dfrac{AB}{BE} = \dfrac{BE}{BC}$, determine the value of $\dfrac{AB}{BC}$.

2. Find the integer m so that

$$10^m < \frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \ldots \frac{99}{100} < 10^{m+1}$$

3. If f is a function such that $f(a+b) = \dfrac{1}{f(a)} + \dfrac{1}{f(b)}$, find all possible values of $f(2011)$.

Answers and Solutions

Qualifying Round

I.

1. D
2. C
3. D
4. B
5. A
6. D
7. B
8. D
9. C
10. D
11. C
12. B
13. D
14. A
15. D

II.

1. C
2. D
3. C
4. D
5. A
6. A
7. B
8. C
9. D
10. C

III.

1. A
2. D
3. C
4. C
5. A

Area Stage

I.

1. $\frac{54}{13}$
2. $\frac{5}{4}$
3. $\frac{20}{3}$
4. $\sqrt{5}$
5. 49
6. 3
7. 59
8. 22940
9. 1014
10. $\frac{1000001}{1000}$
11. $(4, 0)$
12. $(3, 1)$, $\left(-2, -\frac{2}{3}\right)$
13. 4
14. -5
15. 6
16. $5, 1$
17. -1
18. $\frac{4}{5}$
19. $\frac{2}{3}r$
20. 8

II.

1. Let x be BE and y be AE. Note that $AEFD$ is a square so $AE = BC = y$. Also, $AB = BE + AE$ so $AB = x + y$. Since $\dfrac{AB}{BE} = \dfrac{BE}{BC}$ then $\dfrac{x+y}{x} = \dfrac{x}{y}$. Thus, we have, $xy + y^2 = x^2$ which yields to $x^2 - xy - y^2 = 0$. Solving for x using the quadratic formula gives us, $x = \dfrac{y \pm \sqrt{y^2 - 4(1)(-y^2)}}{2} = \left(\dfrac{1 \pm \sqrt{5}}{2}\right) y$. However, we will only take $x = \left(\dfrac{1 + \sqrt{5}}{2}\right) y$ since the other solution will mean that $x < 0$ which is absurd since x is a measure of length. Thus, $\dfrac{AB}{BC} = \dfrac{x+y}{y} = \dfrac{\left(\dfrac{1+\sqrt{5}}{2}\right)y + y}{y} = \dfrac{1 + \sqrt{5} + 2}{2} = \dfrac{3 + \sqrt{5}}{2}$.

   Therefore, the answer is $\boxed{\dfrac{3 + \sqrt{5}}{2}}$.

2. Let $a = \dfrac{1}{2} \times \dfrac{3}{4} \times \dfrac{5}{6} \times \cdots \times \dfrac{99}{100} = \dfrac{3}{2} \times \dfrac{5}{4} \times \dfrac{7}{6} \times \cdots \times \dfrac{99}{98} \times \dfrac{1}{100}$. Hence, $a > \dfrac{1}{100} = 10^{-2}$. Thus, $m \geq -2$.

   Now, let $b = \dfrac{2}{3} \times \dfrac{4}{5} \times \dfrac{6}{7} \times \cdots \times \dfrac{96}{97} \times \dfrac{98}{99}$. Notice that $\dfrac{2}{3} > \dfrac{1}{2}$, $\dfrac{4}{5} > \dfrac{3}{4}$, $\ldots$, $\dfrac{98}{99} > \dfrac{97}{98}$. Also, since $\dfrac{99}{100} < 1$, we have $a < b$. Since $a > 0$ then $a^2 < ab$. But $ab = \dfrac{1}{100}$, so that $a^2 < \dfrac{1}{100}$. Hence, $a < \dfrac{1}{10} = 10^{-1}$. Thus, $m \leq -2$. Therefore, $\boxed{m = -2}$.

3. Consider $f(0) = f(0+0) = \dfrac{1}{f(0)} + \dfrac{1}{f(0)}$ which gives $[f(0)]^2 = 2$. Thus, $f(0) = \pm\sqrt{2}$.
Let $x = f(2011)$.
If $f(0) = \sqrt{2}$ then $x = f(2011) = f(2011+0) = \dfrac{1}{f(2011)} + \dfrac{1}{f(0)} = \dfrac{1}{x} + \dfrac{1}{\sqrt{2}}$. So, $x = \dfrac{\sqrt{2}+x}{\sqrt{2}x}$ which yields to $\sqrt{2}x^2 - x - \sqrt{2} = 0$.
Solving for x using the quadratic formula yields to,

$$x = \frac{1 \pm \sqrt{1 - 4(\sqrt{2})(-\sqrt{2})}}{2\sqrt{2}} = \frac{1 \pm 3}{2\sqrt{2}}$$

Hence, if $f(0) = \sqrt{2}$ then either $f(2011) = \sqrt{2}$ or $f(2011) = -\dfrac{\sqrt{2}}{2}$.
However, suppose $f(2011) = -\dfrac{\sqrt{2}}{2}$. Consider $f(0) = f(2011 + (-2011)) = \dfrac{1}{f(2011)} + \dfrac{1}{f(-2011)}$ which implies that $\sqrt{2} = -\sqrt{2} + \dfrac{1}{f(-2011)}$. Thus, $f(-2011) = \dfrac{\sqrt{2}}{4}$.
But if we consider $f(-2011) = f(-2011+0) = \dfrac{1}{f(-2011)} + \dfrac{1}{f(0)}$, this means that $\dfrac{\sqrt{2}}{4} = 2\sqrt{2} + \dfrac{1}{f(0)}$.
Thus, $f(0) = -\dfrac{2\sqrt{2}}{7}$ which is a contradiction. Thus for $f(0) = \sqrt{2}$, $f(2011) = \sqrt{2}$.
If $f(0) = -\sqrt{2}$ then $x = f(2011) = f(2011+0) = \dfrac{1}{f(2011)} + \dfrac{1}{f(0)} = \dfrac{1}{x} - \dfrac{1}{\sqrt{2}}$.
So, $x = \dfrac{\sqrt{2}-x}{\sqrt{2}x}$ which yields to $\sqrt{2}x^2 + x - \sqrt{2} = 0$.
Solving for x using the quadratic formula yields to,

$$x = \frac{-1 \pm \sqrt{1 - 4(\sqrt{2})(-\sqrt{2})}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}}$$

Hence, if $f(0) = -\sqrt{2}$ then either $f(2011) = -\sqrt{2}$ or $f(2011) = \dfrac{\sqrt{2}}{2}$.
However, suppose $f(2011) = \dfrac{\sqrt{2}}{2}$. Consider $f(0) = f(2011 + (-2011)) = \dfrac{1}{f(2011)} + \dfrac{1}{f(-2011)}$ which implies that $-\sqrt{2} = \sqrt{2} + \dfrac{1}{f(-2011)}$. Thus, $f(-2011) = -\dfrac{\sqrt{2}}{4}$.
But if we consider $f(-2011) = f(-2011+0) = \dfrac{1}{f(-2011)} + \dfrac{1}{f(0)}$, this means that $-\dfrac{\sqrt{2}}{4} = -2\sqrt{2} + \dfrac{1}{f(0)}$.
Thus, $f(0) = \dfrac{2\sqrt{2}}{7}$ which is a contradiction. Thus for $f(0) = -\sqrt{2}$, $f(2011) = -\sqrt{2}$.
So the possible values for $f(2011)$ are $\boxed{\pm\sqrt{2}}$.

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