



18th Philippine Mathematical Olympiad

National Stage, Written Phase

23 January 2016

Time: 4.5 hours

Each item is worth 8 points.

1. The operations below can be applied on any expression of the form $ax^2 + bx + c$.

(I) If $c \neq 0$, replace a by $4a - \frac{3}{c}$ and c by $\frac{c}{4}$.

(II) If $a \neq 0$, replace a by $-\frac{a}{2}$ and c by $-2c + \frac{3}{a}$.

(III) _{t} Replace x by $x - t$, where t is an integer. (Different values of t can be used.)

Is it possible to transform $x^2 - x - 6$ into each of the following by applying some sequence of the above operations?

(a) $5x^2 + 5x - 1$

(b) $x^2 + 6x + 2$

2. Prove that the arithmetic sequence 5, 11, 17, 23, 29, ... contains infinitely many primes.

3. Let n be any positive integer. Prove that

$$\sum_{i=1}^n \frac{1}{(i^2 + i)^{3/4}} > 2 - \frac{2}{\sqrt{n+1}}.$$

4. Two players, A (first player) and B , take alternate turns in playing a game using 2016 chips as follows: *the player whose turn it is, must remove s chips from the remaining pile of chips, where $s \in \{2, 4, 5\}$* . No one can skip a turn. The player who at some point is unable to make a move (cannot remove chips from the pile) loses the game. Who among the two players can force a win on this game?

5. Pentagon $ABCDE$ is inscribed in a circle. Its diagonals AC and BD intersect at F . The bisectors of $\angle BAC$ and $\angle CDB$ intersect at G . Let AG intersect BD at H , let DG intersect AC at I , and let EG intersect AD at J . If $FHGI$ is cyclic and

$$JA \cdot FC \cdot GH = JD \cdot FB \cdot GI,$$

prove that G , F and E are collinear.