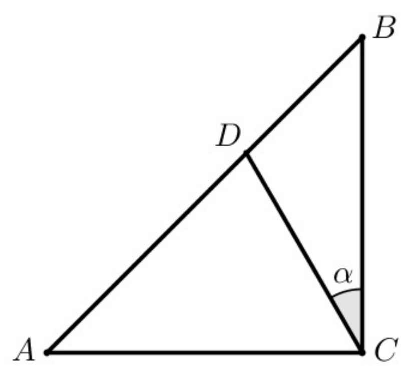
$\boxed{\text{EASY}}$ 15 seconds, 2 points

1. What is the smallest number that is greater than 2015 and divisible by both 6 and 35?

2. A function $f(x)$ satisfies
$$(2-x)f(x) - 2f(3-x) = -x^3 + 5x - 18$$
for all real numbers x. Solve for $f(0)$.

3. Let $f(x) = \ln x$. What are the values of x in the domain of $(f \circ f \circ f \circ f \circ f)(x)$?

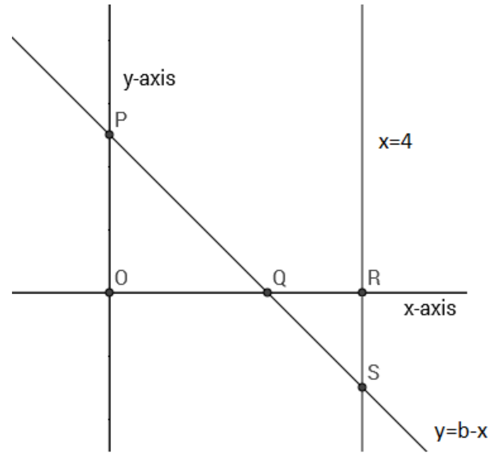
4. There are two distinct real numbers which are larger than their reciprocals by 2. Find the product of these numbers.



5. In the figure on the right, $|AC| = |BC| = 1$ unit, $\alpha = 30°$, and $\angle ACB = 90°$. Find the area of $\triangle ADC$.

6. An urn contains five red chips numbered 1 to 5, five blue chips numbered 1 to 5, and five white chips numbered 1 to 5. Two chips are drawn from this urn without replacement. What is the probability that they have either the same color or the same number?

7. Let n be a positive integer greater than 1. If $2n$ is divided by 3, the remainder is 2. If $3n$ is divided by 4, the remainder is 3. If $4n$ is divided by 5, the remainder is 4. If $5n$ is divided by 6, the remainder is 5. What is the least possible value of n?



8. In the figure on the right, the line $y = b - x$, where $0 < b < 4$, intersects the y-axis at P and the line $x = 4$ at S. If the ratio of the area of $\triangle QRS$ to the area of $\triangle QOP$ is $9 : 25$, determine the value of b.

9. If $\tan x + \tan y = 5$ and $\tan(x + y) = 10$, find $\cot^2 x + \cot^2 y$.

10. Let $\square ABCD$ be a trapezoid with parallel sides AB and CD of lengths 6 units and 8 units, respectively. Let E be the point of intersection of the extensions of the nonparallel sides of the trapezoid. If the area of $\triangle BEA$ is 60 square units, what is the area of $\triangle BAD$?

11. How many solutions does the equation $x + y + z = 2016$ have, where x, y and z are integers with $x > 1000$, $y > 600$, and $z > 400$?

12. Find all values of integers x and y satisfying $2^{3x} + 5^{3y} = 189$.

13. In parallelogram $ABCD$, $\angle BAD = 76°$. Side AD has midpoint P, and $\angle PBA = 52°$. Find $\angle PCD$.

14. Find the smallest number k such that for all real numbers x, y and z

$$(x^2 + y^2 + z^2)^2 \leq k(x^4 + y^4 + z^4).$$

15. Last January 7, 2016, a team from the University of Central Missouri headed by Curtis Cooper discovered the largest prime number known so far:

$$2^{74,207,281} - 1$$

which contains over 22.3 million digits. Curtis Cooper is part of a large collaborative project called GIMPS, where mathematicians use their computers to look for prime numbers of the form 1 less than a power of 2. What is the meaning of GIMPS?

$\boxed{\text{AVERAGE}}$ 30 seconds, 3 points

1. Find the value of $\cot\left(\cot^{-1} 2 + \cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5\right)$.

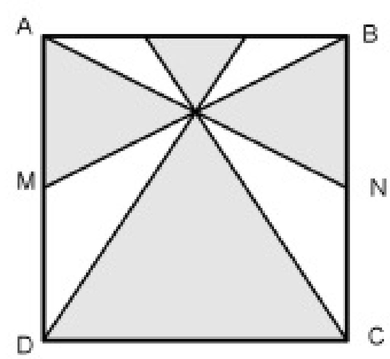
2. Find the minimum value of $x^2 + 4y^2 - 2x$, where x and y are real numbers that satisfy $2x + 8y = 3$.

3. Alice, Bob, Charlie and Eve are having a conversation. Each of them knows who are honest and who are liars. The conversation goes as follows:

   Alice: Both Eve and Bob are liars.
   Bob: Charlie is a liar.
   Charlie: Alice is a liar.
   Eve: Bob is a liar.

   Who is/are honest?

4. Let $f(x)$ be a polynomial function of degree 2016 whose 2016 zeroes have a sum of S. Find the sum of the 2016 zeroes of $f(2x - 3)$ in terms of S.



5. Refer to the figure on the right. The quadrilateral $ABCD$ is a square with a side of length 2 units while M and N are the midpoints of AD and BC, respectively. Determine the area of the shaded region.

6. Suppose that Ethan has four red chips and two white chips. He selects three chips at random and places them in Urn 1, while the remaining chips are placed in Urn 2. He then lets his brother Josh draw one chip from each urn at random. What is the probabiity that the chips drawn by Josh are both red?

7. Let $f(x)$ be a function such that $f(1) = 1$, $f(2) = 2$ and $f(x+2) = f(x+1) - f(x)$. Find $f(2016)$.

8. In a certain school, there are 5000 students. Each student is assigned an ID number from 0001 to 5000. No two students can have the same ID number. If a student is selected uniformly at random, what is the probability that the ID number of the student does not contain any 2s among its digits?

9. 120 unit cubes are put together to form a rectangular prism whose six faces are then painted. This leaves 24 unit cubes without any paint. What is the surface area of the prism?

10. Let m be the product of all positive integer divisors of $360,000$. Suppose the prime factors of m are $p_1, p_2, \ldots, p_k$, for some positive integer k, and $m = p_1^{e_1} p_2^{e_2} \cdot \ldots \cdot p_k^{e_k}$, for some positive integers e_1, $e_2, \ldots, e_k$. Find $e_1 + e_2 + \ldots + e_k$.

$\boxed{\text{DIFFICULT}}$ 60 seconds, 6 points

1. The irrational number 0.123456789101112... is formed by concatenating, in increasing order, all the positive integers. Find the sum of the first 2016 digits of this number after the decimal point.

2. Suppose $\frac{1}{2} \leq x \leq 2$ and $\frac{4}{3} \leq y \leq \frac{3}{2}$. Determine the minimum value of

$$\frac{x^3y^3}{x^6 + 3x^4y^2 + 3x^3y^3 + 3x^2y^4 + y^6}.$$

3. In an $n \times n$ checkerboard, the rows are numbered 1 to n from top to bottom, and the columns are numbered 1 to n from left to right. Chips are to be placed on this board so that each square has a number of chips equal to the absolute value of the difference of the row and column numbers. If the total number of chips placed on the board is 2660, find n.

4. $ABCD$ is a cyclic quadrilateral such that $|DA| = |BC| = 2$, and $|AB| = 4$. If $|CD| > |AB|$ and the lines DA and BC intersect at an angle of $60°$, find the radius of the circumscribing circle.

5. The faces of a 12-sided die are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 such that the sum of the numbers on opposite faces is 13. The die is meticulously carved so that it is biased: the probability of obtaining a particular face F is greater than 1/12, the probability of obtaining the face opposite F is less than 1/12 while the probability of obtaining any one of the other ten faces is 1/12. When two such dice are rolled, the probability of obtaining a sum of 13 is 29/384. What is the probability of obtaining face F?

$\boxed{\text{SPARE}}$ 30 seconds, 3 points

1. Inside square $ABCD$, a point E is chosen so that triangle DEC is equilateral. Find the measure of $\angle AEB$.

2. Find all triples of positive real numbers (x, y, z) which satisfy the system

$$\begin{cases} \sqrt[3]{x} - \sqrt[3]{y} - \sqrt[3]{z} = 64 \\ \sqrt[4]{x} - \sqrt[4]{y} - \sqrt[4]{z} = 32 \\ \sqrt[6]{x} - \sqrt[6]{y} - \sqrt[6]{z} = 8 \end{cases}$$

3. Find the minimum value of $x^2 + 4y^2 - 2x$, where x and y are real numbers that satisfy $2x + 8y = 3$.

4. Find all real numbers x that satisfies

$$\frac{x^4 + x + 1}{x^4 + x^2 - x - 4} = \frac{x^4 + 1}{x^4 + x^2 - 4}.$$

5. Find all positive real numbers a, b, c, d such that for all $x \in \mathbb{R}$,

$$(ax + b)^{2016} + (x^2 + cx + d)^{1008} = 8(x - 2)^{2016}.$$

6. How many different integral solutions (x, y) does $3|x| + 5|y| = 100$ have?

7. An ant situated at point A decides to walk 1 foot east, then $\frac{1}{2}$ foot northeast, then $\frac{1}{4}$ foot east, then $\frac{1}{8}$ foot northeast, then $\frac{1}{16}$ foot east and so on (that is, the ant travels alternately between east and northeast, and the distance travelled is decreased by half every time the ant changes its direction). The ant eventually reaches a certain point B. Determine the distance between the ant's unitial and final positions.

8. Find the last two digits of 2^{100}.

9. Let $f(x) = 2^x - 2^{1-x}$. Simplify $\sqrt{f(2015) - f(2014) + f(1) - f(0)}$.

   Answer: $2^{1007} + 2^{-1007}$

10. A school program will randomly start between 8:30AM and 9:30AM and will randomly end between 7:00PM and 9:00PM. What is the probability that the program lasts for at least 11 hours and starts before 9:00AM?

11. Find the last three digits of $2016^3 + 2017^3 + 2018^3 + \ldots + 3014^3$.