about the pmo

First held in 1984, the PMO was created as a venue for high school students with interest and talent in mathematics to come together in the spirit of friendly competition and sportsmanship. Its aims are: (1) to awaken greater interest in and promote the appreciation of mathematics among students and teachers; (2) to identify mathematically-gifted students and motivate them towards the development of their mathematical skills; (3) to provide a vehicle for the professional growth of teachers; and (4) to encourage the involvement of both public and private sectors in the promotion and development of mathematics education in the Philippines.

The PMO is the first part of the selection process leading to participation in the International Mathematical Olympiad (IMO). It is followed by the Mathematical Olympiad Summer Camp (MOSC), a five-phase program for the twenty national finalists of PMO. The four selection tests given during the second phase of MOSC determine the tentative Philippine Team to the IMO. The final team is determined after the third phase of MOSC.

The PMO this year is the fifteenth since 1984. Almost two thousand four hundred (2400) high school students from all over the country took the qualifying examination, out of these, two hundred twelve (212) students made it to the Area Stage. Now, in the National Stage, the number is down to twenty and these twenty students will compete for the top three positions and hopefully move on to represent the country in the 55th IMO, which will be held in Cape Town, South Africa on July 3-13, 2014.

message from dost-sei

Our winning streak in the International Mathematics Olympiad (IMO), widely acknowledged as the hardest mathematics competition in the world, is a statement to the increased level of competition in the Philippine Mathematical Olympiad (PMO). As a national showcase of math prowess among our most gifted students, it has succeeded in producing not just medalists to the IMO, but icons that can inspire the even younger generation to follow their footsteps in the field.

This era has been a challenging one to live in as the impacts of the changing climate become more extreme and more frequent. While it is alarming, we should be encouraged by the fact that consciousness in science and technology among our people is becoming high. The fact that you, at an early age, are excelling in this prestigious competition makes us believe that we are developing the next breed of leaders that the present and future generations will rely on in terms of providing S&T-based solutions to major issues.

We truly believe that Filipino students, given the proper support and encouragement, can make it big as leaders in the future.

We are optimistic that through the PMO, our search for the top talents in science and mathematics would always be fruitful. We hope that our participants will dedicate their gifts in service of the people through science and mathematics.

We look forward to an exciting PMO and we wish all the contestants the best.

FORTUNATO T. DE LA PEÑA
Undersecretary for S&T Services, DOST and
Officer-in-Charge, SEI

message from dep-ed

Congratulations to the participants of this year's Mathematical Olympiad!

For 16 years, the Mathematical Olympiad has been the avenue for honoring the analytical minds of the country's brightest. Competitions like these complement the technical information we teach our learners in the formal school setting.

We must remember that information learned better serves its purpose when is translated to real-life application. This competition is not only a test of how the students mastered given information, but an assessment of how much students have accustomed themselves to the discipline of rigor training and critical thinking.

With tournaments like this, you help the Department achieve its present and long-term goals. May we keep on working together to fulfill our one goal: to keep our learners enflamed with the passion for learning.

BR. ARMIN A. LUSTRO FSC

Secretary

Department of Education

message from msp

The aim of the Philippine Mathematical Olympiad (PMO) is to identify and reward excellence in Mathematics. We hope to discover and nurture talents and hopefully attract them to careers in Science and Mathematics in the future. We are grateful to the Department of Science and Technology-Science Education Institute (DOST-SEI) for partnering with us in organizing this activity. The MSP and DOST-SEI both believe that competitions enhance education.

MSP is proud to organize the PMO, the toughest and most prestigious math competition in the country. Congratulations to the winners and all the participants of the 16th PMO! They have displayed good Filipino values such as determination, hard work and optimism.

The School Year 2013-14 has been a challenging year for our country most especially to our friends in the Visayas and Mindanao. The organization of the PMO was not exempt from the difficulties brought by the disasters. May the challenges brought by the earthquake in Cebu and Bohol and the typhoon "Yolanda" in some parts of the Visayas inspire us more to do our share so that our country can move forward in overcoming these tragedies.

In behalf of the MSP, I wish to thank the sponsors, schools and other organizations, institutions and individuals for their continued support and commitment to the PMO. Thank you and congratulations to Dr. Richard Lemence and his team for the successful organization of the 16th PMO.

Jumela Sarmiento

Jumela F. Sarmiento, Ph.D.
President
Mathematical Society of the Philippines

message from fuse

I am very pleased to hear of the upcoming activities of the 16th Philippine Mathematical Olympiad (PMO). To me Mathematics is such a challenging subject in school and everywhere. As such students' interest in it should be sustained and nurtured.

Congratulations to the MSP for what it has been along this line. What you have been doing is truly noteworthy.

May you have more Philippine Mathematics Olympiads to help recognize and nurture mathematical talents in our country.

LUCIO C. TAN

Vice-Chairman

FUSE

message from c&e

I write this message as I listen to news about the Philippine Azkals Football team preparing for a fight with Spanish players coming very soon to the Philippines. The celebrity status achieved by the Azkals members is an indication of how promising this re-discovered sports is to Filipinos. Here is finally a sports where people, regardless of height or country of origin can excel.

I would like to think of Mathematics as the football of high school subjects and the PMO as the Azkals of international scholastic competitions.

Mathematics as a subject does level the playing field and the success of Filipino students in this subject when competing abroad is testimony to how the Philippines can keep earning another reputation for being home to world-class Math champions.

In keeping with my personal belief that we indeed have the best Math students this side of the planet, rest assured that C&E Publishing, Inc. will always be behind the Philippine Math Olympiad in the Organization's noble quest to produce the brightest of young mathematicians.

Congratulations to all the qualifiers to the National Level. Congratulations to the members and officers of the Philippine Math Olympiad for once again staging and now having the 15th Philippine Mathematical Olympiad.

May your effort keep on exponentially multiplying into the highest Mersenne prime possible. Mabuhay!

EMYL EUGENIO
VP-Sales and Marketing Division
C&E Publishing, Inc.

message from sharp

Let me first congratulate the Mathematical Society of the Philippines for their extra efforts in providing world class Filipino students in gearing towards mathematics excellence.

We from Collins International, distributor of world class brands like Sharp Calculators, are very proud to be part of this worthy project. Philippine Math Olympiad program brings prestige to every participant and their families as well.

Once again, six finalists of this year's PMO Top 20 performers will represent the Philippines in International Math Olympiad. We pray the good Lord Jesus will bless them with greater knowledge to bring home the gold.

SHARP
CALCULATORS

LUCERO ONG
Assistant Vice-President
Sharp Calculators
Collins International Trading Corporation

the pmo team

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Region 11
Dr Eveyth Deligero

Region 12 / ARMM
Dr Jonald Pimentel

Region 13
Dr Thelma Montero-Galliguez

NCR
Dr Recto Rex Calingasan

Schedule

0800AM - 0830AM	Registration
	Br Andrew Gonzalez (Bag) Bldg
	Lobby
0900AM - 1200NN	Phase I: Written Phase
	12th Fl. (A1210), BAG Building
1200NN - 0140PM	Lunch Break
0140PM - 0200PM	Oral Phase Orientation of
	Finalists
0200PM - 0500PM	Phase II: Oral Phase
0630PM - 0830PM	Dinner & Awarding Ceremonies
	4th Fl. Henry Sy Sr. Hall

the pmo finalists

Clyde Wesly Si Ang
Chiang Kai Shek College
Coach: Armi R. Mogro

John Angel Libranda Aranas
Makati Science High School
Coach: Mark Anthony J. Vidallo

John Thomas Yu Chuatak
St. Stephen's High School
Coach: Tom Ng Chu

Kyle Patrick Flor Dulay
Philippine Science HS Main
Coach: Jose Manresa Enrico D. Espanol IV

Raymond Joseph Fadri
Makati Science High School
Coach: Arnel D. Olofernes

Andrea Jaba
St. Jude Catholic School
Coach: Manuel Tanpoco

Ma. Czarina Angela Sio Lao
St. Jude Catholic School
Coach: Manuel Tanpoco

Tiong Soon Kelsey Lim
Grace Christian College
Coach: John Frederick T. Soriano

Hans Rhenzo Manguiat
Ateneo de Manila High School
Coach: Marvin Coronel

Jose Alfonso Mirabueno
Philippine Science HS Southern Mindanao
Coach: Harold C. Soriano

Andrew Brandon Ong
Chiang Kai Shek College
Coach: Monnette E. Defeo

Gerald Pascua
Philippine Science HS Main
Coach: Jose Manresa Enrico D. Espanol IV

Albert John Patupat
Holy Rosary College
Coach: Nacymova A. Magat

Shaquille Wyan Que
Grace Christian College
Coach: John Frederick T. Soriano

Reine Jiana Mendoza Reynoso
Philippine Science HS Main
Coach: Jose Manresa Enrico D. Espanol IV

Immanuel Gabriel Sin
Ateneo de Manila High School
Coach: Marvin Coronel

Adrian Reginald Chua Sy
St. Jude Catholic School
Coach: Manuel Tanpoco

Matthew Sy Tan
St. Jude Catholic School
Coach: Manuel Tanpoco

Farrell Eldrian So Wu
MGC New Life Christian Academy
Coach: Neshie Joyce Guntiñas

Kaye Janelle Yao
Grace Christian College
Coach: John Frederick T. Soriano

pmo: through the years



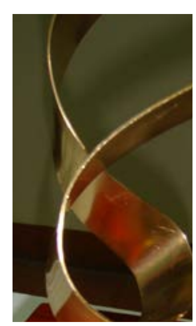












16^{th} Philippine Mathematical Olympiad

Qualifying Stage

12 October 2013

PART I. Each correct answer is worth two points.

1. Simplify $\frac{5}{6} - \frac{7}{12} + \frac{9}{20} - \frac{11}{30} + \frac{13}{42} - \frac{15}{56}$.

   (a) $\frac{3}{8}$ (b) $\frac{7}{12}$ (c) $\frac{5}{14}$ (d) $\frac{9}{14}$

2. Brad and Angelina each tipped their waiter 50 pesos. Brad tipped 4% of his bill while Angelina tipped 10% of her bill. What is the total bill of the two?

   (a) 1500 pesos (b) 1750 pesos (c) 1250 pesos (d) 2250 pesos

3. What is the probability of getting a sum of 10 when rolling three fair six-sided dice?

   (a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

4. Eighteen 1 cm $\times$ 1 cm square tiles are arranged to form a rectangle, with no overlaps and without leaving gaps in the interior. Which of the following is NOT a possible perimeter of the rectangle?

   (a) 18 cm (b) 22 cm (c) 24 cm (d) 38 cm

5. Consider the sum $S = x! + \sum_{i=0}^{2013} i!$, where x is a one-digit nonnegative integer. How many possible values of x are there so that S is divisible by 4?

   (a) 2 (b) 3 (c) 4 (d) 5

6. Find the number of integer solutions less than 5 that satisfy the inequality $(x^3 + 4x^2)(x^2 - 3x - 2) \leq (x^3 + 4x^2)(2x^2 - 6)$.

   (a) 2 (b) 3 (c) 4 (d) 5

7. For which m does the equation $\dfrac{x-1}{x-2} = \dfrac{x-m}{x-6}$ have no solution in x?

(a) 6 (b) 2 (c) 8 (d) 5

8. What is the remainder when 2^{201} is divided by 7?

(a) 1 (b) 4 (c) 2 (d) 3

9. Let $b_0 = 2$ and $b_1 = 1$, and $b_{n+1} = b_{n-1} + b_n$. Which of the following digits is the last to appear in the units position of the sequence $\{b_n\}$?

(a) 6 (b) 7 (c) 9 (d) 0

10. $3x - b = 1$ and $bx - 5 = -4$ have the same positive solution x. Find b.

(a) $\dfrac{-1+\sqrt{13}}{2}$ (c) $\dfrac{1+\sqrt{13}}{2}$

(b) $\dfrac{-1+\sqrt{13}}{6}$ (d) $\dfrac{1+\sqrt{13}}{6}$

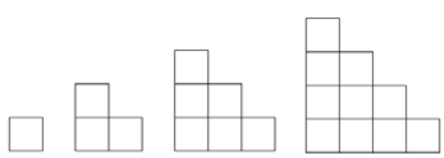
11. If all the words (meaningful or meaningless) obtained from permuting the letters of the word SMART are arranged alphabetically, what is the rank of the word SMART?

(a) 72nd (b) 79th (c) 80th (d) 78th

12. Evaluate $\sqrt[3]{\sqrt{32}}\sqrt[4]{\sqrt[3]{32}}\sqrt[5]{\sqrt[4]{32}}\cdots\sqrt[10]{\sqrt[9]{32}}$.

(a) 4 (b) 8 (c) 2 (d) 16

13. The figures below, consisting of unit squares, are the first four in a sequence of "staircases," How many unit squares are there in the 2013th staircase?



(a) 2014×1007 (b) 2013×1007 (c) 2013×1008 (d) 2013×2007

14. How many factors of 7^{9999} are greater than 1000000?

(a) 9989 (b) 9990 (c) 9991 (d) 9992

15. Let $\overline{AB}$ be a chord of circle C with radius 13. If the shortest distance of $\overline{AB}$ to point C is 5, what is the perimeter of ΔABC?

(a) 30 units (b) 60 units (c) 50 units (d) 25 units

PART II. Each correct answer is worth three points.

1. Find the range of the function $f(x) = \dfrac{4^{x+1} - 3}{4^x + 1}$

(a) $(-3, 1)$ (c) $(-3, 4)$

(b) $(-4, 3)$ (d) $(4, +\infty)$

2. If k consecutive integers sum to -1, find the sum of the largest and smallest term.

(a) -1 (b) 0 (c) 1 (d) k

3. If $2xy + y = 43 + 2x$ for positive integers x, y, find the largest value of $x + y$.

(a) 10 (b) 13 (c) 14 (d) 17

4. Let (a, b) and (c, d) be two points of the circles C_1 and C_2. The circle C_1 is centered at the origin and passes through $P(16, 16)$, while the circle C_2 is centered at P and passes through the origin. Find $a + b + c + d$.

(a) 16 (b) 32 (c) $16\sqrt{2}$ (d) $\dfrac{32}{\sqrt{3}}$

5. Let f be a function that satisfies $f(x + y) = f(x)f(y)$ and $f(xy) = f(x) + f(y)$ for all real numbers x, y. Find $f(\pi^{2013})$.

(a) 2013 (b) 0 (c) 1 (d) π

6. Evaluate $\log_2 \sin(\pi/8) + \log_2 \cos(15\pi/8)$.

(a) $1/2$ (b) 0 (c) -1 (d) $-3/2$

7. If the sum of the infinite geometric series $\frac{a}{b} + \frac{a}{b^2} + \frac{a}{b^3} + \cdots$ is 4, then what is the sum of

$$\frac{a}{a+b} + \frac{a}{(a+b)^2} + \frac{a}{(a+b)^3} + \cdots ?$$

(a) 0.8 (b) 1 (c) 1.2 (d) 1.6

8. How many trailing zeros does 126! have when written in decimal notation?

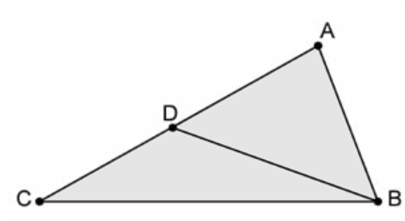
(a) 30 (b) 31 (c) 25 (d) 12

9. It is known that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}$. Find the sum

$$1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots .$$

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{9}$ (c) $\frac{\pi^2}{10}$ (d) $\frac{\pi^2}{12}$

10. In ΔABC, a point D is on $\overline{AC}$ so that $AB = AD$ so that $m\angle ABC - m\angle ACB = 45°$. Find $m\angle CBD$.



(a) 15° (b) 18° (c) 22.5° (d) 30°

PART III. Each correct answer is worth six points.

1. Let a_n be a sequence such that the average of the first and second terms is 1, the average of the second and third terms is 2, the average of the third and fourth terms is 3, and so on. Find the average of the 1st and 100th terms.

(a) 75 (b) 25 (c) 50 (d) 100

2. Let f be a function such that $f(0) = 1$ and

$$f(2xy - 1) = f(x)f(y) - f(x) - 2y - 1$$

for all x and y. Which of the following is true?

(a) $f(x) \geq 0$ for all real x.
(b) $f(5)$ is a composite number.
(c) $f(7)$ is an even integer.
(d) $f(12)$ is a perfect square.

3. If $\dfrac{a}{a^2+1} = \dfrac{1}{3}$, determine $\dfrac{a^3}{a^6+a^5+a^4+a^3+a^2+a+1}$.

(a) $\dfrac{1}{25}$ (b) $\dfrac{1}{29}$ (c) $\dfrac{1}{33}$ (d) $\dfrac{1}{36}$

4. If $m^3 - 12mn^2 = 40$ and $4n^3 - 3m^2n = 10$, find $m^2 + 4n^2$.

(a) $6\sqrt[3]{2}$ (b) $8\sqrt[3]{2}$ (c) $9\sqrt[3]{2}$ (d) $10\sqrt[3]{2}$

5. Find the minimum value of $2a^8 + 2b^6 + a^4 - b^3 - 2a^2 - 2$, where a and b are real numbers.

(a) $3/8$ (b) $5/8$ (c) $-11/4$ (d) $-11/8$

Answers:

PART 1 (1) A (2) B (3) B (4) C (5) A (6) C (7) D (8) A (9) A (10) A (11) B (12) D (13) B (14) D (15) C

PART 2 (1) C (2) A (3) C (4) B (5) B (6) D (7) A (8) C (9) A (10) C

PART 3 (1) C (2) D (3) B (4) D (5) C

16^{th} Philippine Mathematical Olympiad

Area Stage

7 December 2013

Part I. No solution is needed. All answers must be in simplest form. Each correct answer is worth three points.

1. Find the number of ordered triples (x, y, z) of positive integers satisfying $(x+y)^z = 64$.
2. What is the largest number of $7\,\text{m} \times 9\,\text{m} \times 11\,\text{m}$ boxes that can fit inside a box of size $17\,\text{m} \times 37\,\text{m} \times 27\,\text{m}$?
3. Let $N = (1 + 10^{2013}) + (1 + 10^{2012}) + \cdots + (1 + 10^1) + (1 + 10^0)$. Find the sum of the digits of N.
4. The sequence $2, 3, 5, 6, 7, 8, 10, 11, \ldots$ is an enumeration of the positive integers which are not perfect squares. What is the 150th term of this sequence?
5. Let $P(x) = 1 + 8x + 4x^2 + 8x^3 + 4x^4 + \cdots$ for values of x for which this sum has finite value. Find $P(1/7)$.
6. Find all positive integers m and n so that for any x and y in the interval $[m, n]$, the value of $\dfrac{5}{x} + \dfrac{7}{y}$ will also be in $[m, n]$.
7. What is the largest positive integer k such that $27!$ is divisible by 2^k?
8. For what real values of p will the graph of the parabola $y = x^2 - 2px + p + 1$ be on or above that of the line $y = -12x + 5$?
9. Solve the inequality $\log\left(5^{\frac{1}{x}} + 5^3\right) < \log 6 + \log 5^{1+\frac{1}{2x}}$.
10. Let p and q be positive integers such that $pq = 2^3 \cdot 5^5 \cdot 7^2 \cdot 11$ and $\dfrac{p}{q} = 2 \cdot 5 \cdot 7^2 \cdot 11$. Find the number of positive integer divisors of p.
11. Let r be some real constant, and $P(x)$ a polynomial which has remainder 2 when divided by $x - r$, and remainder $-2x^2 - 3x + 4$ when divided by $(2x^2 + 7x - 4)(x - r)$. Find all values of r.
12. Suppose $\alpha, \beta \in (0, \pi/2)$. If $\tan\beta = \dfrac{\cot\alpha - 1}{\cot\alpha + 1}$, find $\alpha + \beta$.
13. How many positive integers, not having the digit 1, can be formed if the product of all its digits is to be 33750?
14. Solve the equation $(2 - x^2)^{x^2 - 3\sqrt{2}x + 4} = 1$.
15. Rectangle $BRIM$ has $BR = 16$ and $BM = 18$. The points A and H are located on IM and BM, respectively, so that $MA = 6$ and $MH = 8$. If T is the intersection of BA and IH, find the area of quadrilateral $MATH$.

16. Two couples and a single person are seated at random in a row of five chairs. What is the probability that at least one person is not beside his/her partner?

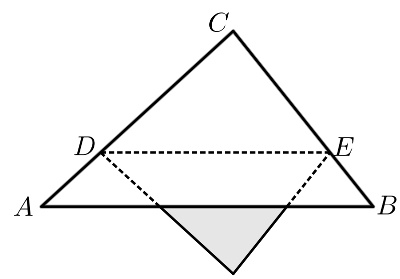
17. Trapezoid $ABCD$ has parallel sides AB and CD, with BC perpendicular to them. Suppose $AB = 13$, $BC = 16$ and $DC = 11$. Let E be the midpoint of AD and F the point on BC so that EF is perpendicular to AD. Find the area of quadrilateral $AEFB$.

18. Let x be a real number so that $x + \dfrac{1}{x} = 3$. Find the last two digits of $x^{2^{2013}} + \dfrac{1}{x^{2^{2013}}}$.

19. Find the values of x in $(0, \pi)$ that satisfy the equation

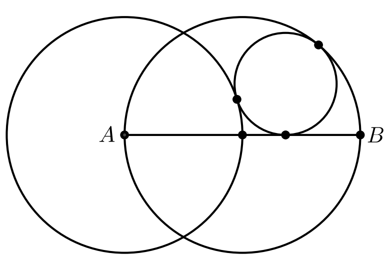
$$(\sqrt{2014} - \sqrt{2013})^{\tan^2 x} + (\sqrt{2014} + \sqrt{2013})^{-\tan^2 x} = 2(\sqrt{2014} - \sqrt{2013})^3.$$

20. The base AB of a triangular piece of paper ABC is 16 cm long. The paper is folded down over the base, with the crease DE parallel to the base of the paper, as shown. The area of the triangle that projects below the base (shaded region) is 16% that of the area of $\triangle ABC$. What is the length of DE, in cm?



Part II. Show the solution to each item. Each complete and correct solution is worth ten points.

1. Two circles of radius 12 have their centers on each other. As shown in the figure, A is the center of the left circle, and AB is a diameter of the right circle. A smaller circle is constructed tangent to AB and the two given circles, internally to the right circle and externally to the left circle, as shown. Find the radius of the smaller circle.



2. Let a, b and c be positive integers such that $\dfrac{a\sqrt{2013} + b}{b\sqrt{2013} + c}$ is a rational number. Show that $\dfrac{a^2 + b^2 + c^2}{a + b + c}$ and $\dfrac{a^3 - 2b^3 + c^3}{a + b + c}$ are both integers.

3. If p is a real constant such that the roots of the equation $x^3 - 6px^2 + 5px + 88 = 0$ form an arithmetic sequence, find p.

16th Philippine Mathematical Olympiad

Area Stage

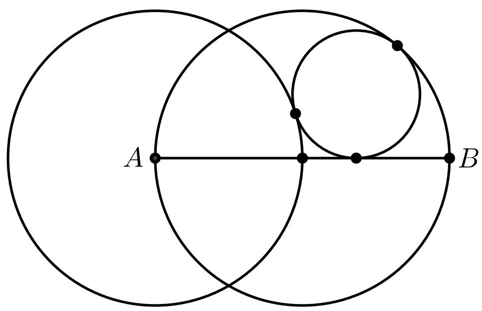
7 December 2013

Part I. No solution is needed. All answers must be in simplest form. Each correct answer is worth three points.

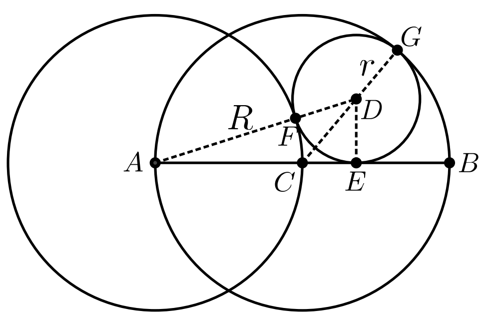
1. 74
2. 18
3. 2021
4. 162
5. $\frac{9}{4} = 2.25$
6. $(m, n) = (1, 12), (2, 6), (3, 4)$
7. 23
8. $5 \leq p \leq 8$
9. $\frac{1}{4} < x < \frac{1}{2}$
10. 72
11. $r = \frac{1}{2}, -2$
12. $\frac{\pi}{4}$
13. 625
14. $\pm 1, 2\sqrt{2}$
15. 34
16. $\frac{2}{5} = 0.4$
17. 91
18. 07
19. $x = \frac{\pi}{3}, \frac{2\pi}{3}$
20. 11.2 cm

Part II. Show the solution to each item. Each complete and correct solution is worth ten points.

1. Two circles of radius 12 have their centers on each other. As shown in the figure, A is the center of the left circle, and AB is a diameter of the right circle. A smaller circle is constructed tangent to AB and the two given circles, internally to the right circle and externally to the left circle, as shown. Find the radius of the smaller circle.



Solution:



Let R be the common radius of the larger circles, and r that of the small circle. Let C and D be the centers of the right large circle and the small circle, respectively. Let E, F and G be the points of tangency of the small circle with AB, the left large circle, and the right large circle, respectively. Since the centers of tangent circles are collinear with the point of tangency, then A-F-D and C-D-G are collinear.

From $\triangle AED$, $AE^2 = (R+r)^2 - r^2 = R^2 + 2Rr$. Therefore, $CE = AE - R = \sqrt{R^2 + 2Rr} - R$.

From $\triangle CED$, $CE^2 = (R-r)^2 - r^2 = R^2 - 2Rr$.

Therefore, $\sqrt{R^2 + 2Rr} - R = \sqrt{R^2 - 2Rr}$. Solving this for r yields $r = \frac{\sqrt{3}}{4}R$. With $R = 12$, we get $r = 3\sqrt{3}$.

2. Let a, b and c be positive integers such that $\dfrac{a\sqrt{2013}+b}{b\sqrt{2013}+c}$ is a rational number. Show that $\dfrac{a^2+b^2+c^2}{a+b+c}$ and $\dfrac{a^3-2b^3+c^3}{a+b+c}$ are both integers.

Solution:

By rationalizing the denominator, $\dfrac{a\sqrt{2013}+b}{b\sqrt{2013}+c} = \dfrac{2013ab-bc+\sqrt{2013}(b^2-ac)}{2013b^2-c^2}$. Since this is rational, then $b^2-ac=0$. Consequently,

$$\begin{aligned} a^2+b^2+c^2 &= a^2+ac+c^2 = (a+c)^2-ac = (a+c)^2-b^2 \\ &= (a-b+c)(a+b+c) \end{aligned}$$

and

$$\begin{aligned} a^3-2b^3+c^3 &= a^3+b^3+c^3-3b^3 = a^3+b^3+c^3-3abc \\ &= (a+b+c)(a^2+b^2+c^2-ab-bc-ca). \end{aligned}$$

Therefore,

$$\frac{a^2+b^2+c^2}{a+b+c} = a-b+c \qquad \text{and} \qquad \frac{a^3-2b^3+c^3}{a+b+c} = a^2+b^2+c^2-ab-bc-ca$$

are integers.

3. If p is a real constant such that the roots of the equation $x^3-6px^2+5px+88=0$ form an arithmetic sequence, find p.

Solution: Let the roots be $b-d$, b and $b+d$. From Vieta's formulas,

$$-88 = (b-d)b(b+d) = b(b^2-d^2) \tag{1}$$
$$5p = (b-d)b+b(b+d)+(b+d)(b-d) = 3b^2-d^2 \tag{2}$$
$$6p = (b-d)+b+(b+d) = 3b \tag{3}$$

From (3), $b=2p$. Using this on (1) and (2) yields $-44=p(4p^2-d^2)$ and $5p=12p^2-d^2$. By solving each equation for d^2 and equating the resulting expressions, we get $4p^2+\frac{44}{p}=12p^2-5p$. This is equivalent to $8p^3-5p^2-44=0$. Since $8p^3-5p^2-44=(p-2)(8p^2+11p+22)$, and the second factor has negative discriminant, we only have $p=2$.

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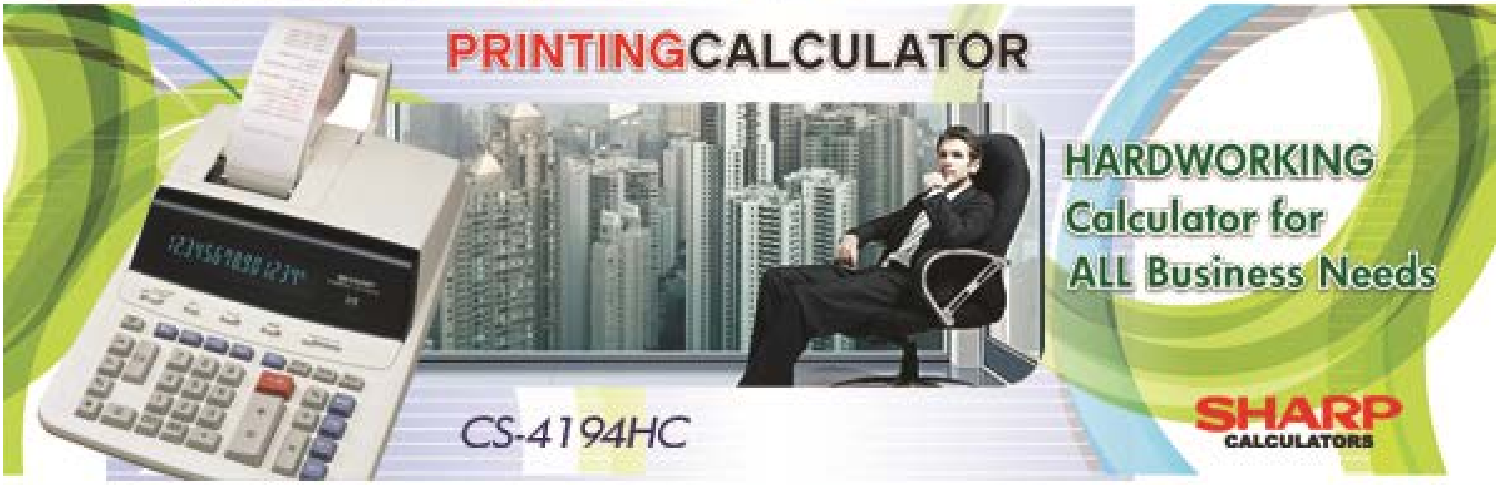

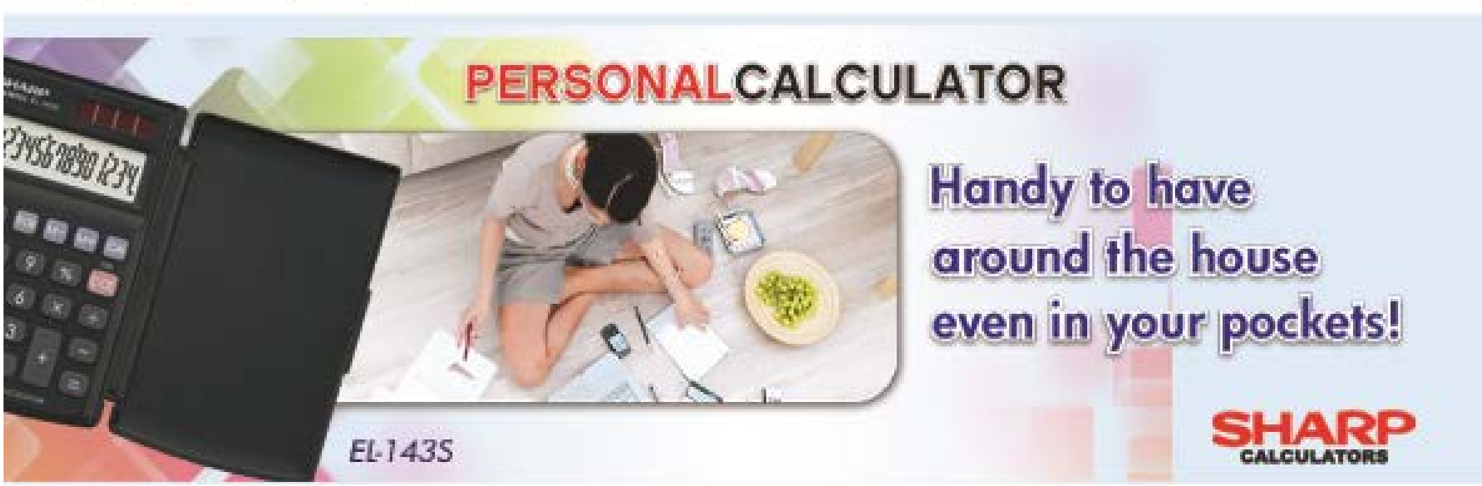

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